CE5510 Advanced Structural Concrete Design
- Design & Detailing of Openings in RC Flexural Members-

Assoc Prof Tan Kiang Hwee
Department of Civil Engineering
National University of Singapore

In this lecture

We will explore

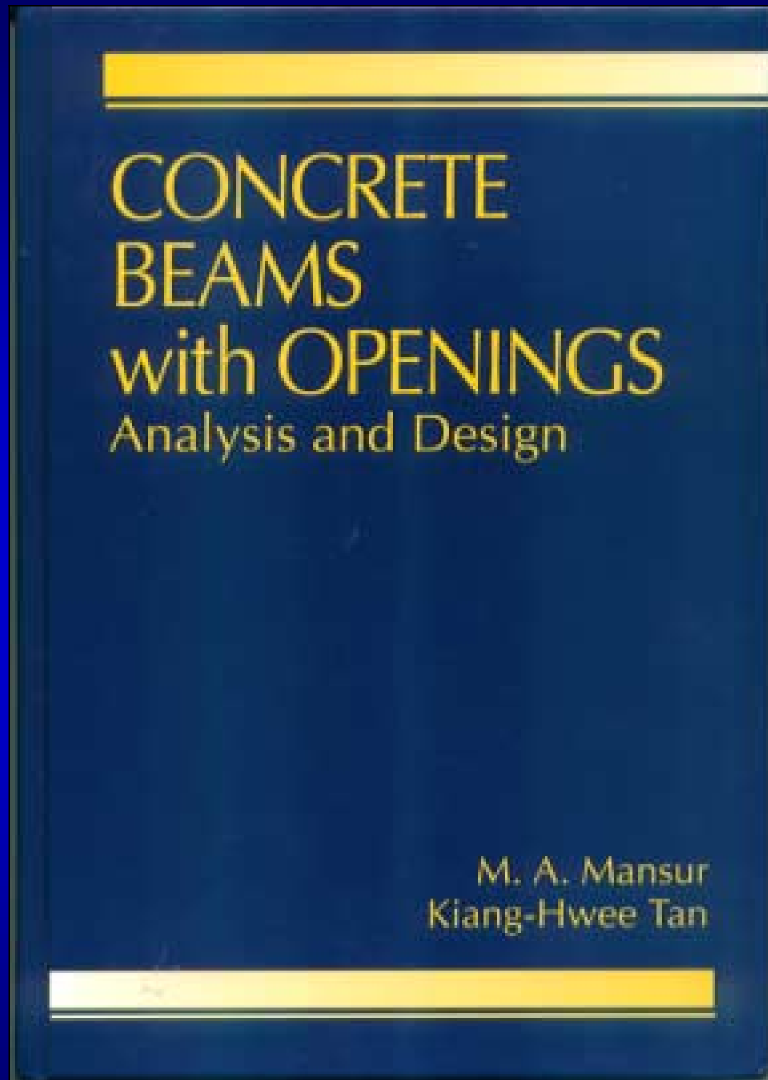
- ❖Types of openings
- ❖Behaviour of beams with large openings
- ❖Design for ultimate strength
- ❖Crack Control
- ❖Deflection Calculation

A/P Tan K H, NUS

At the end of the lecture

You should be able to

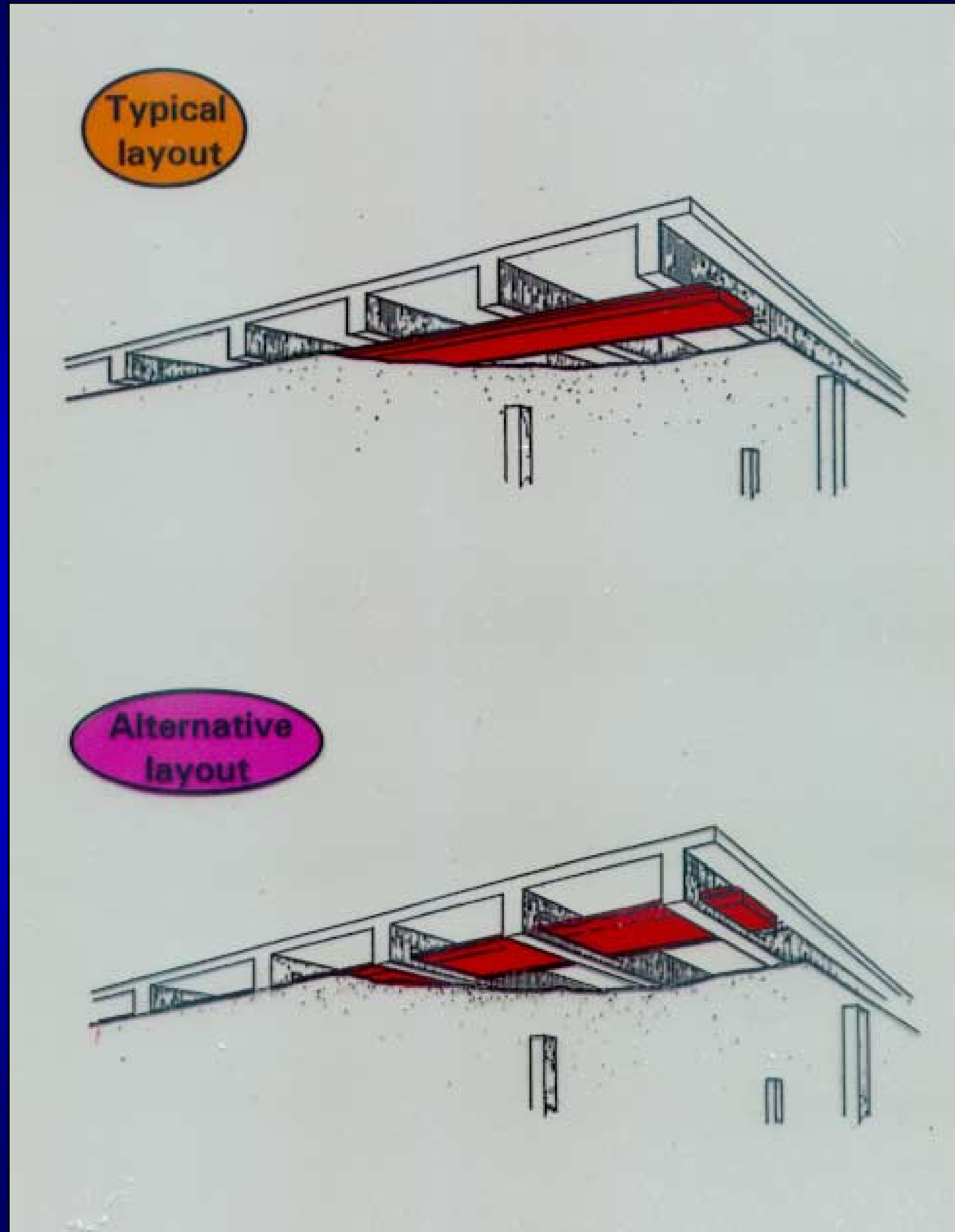
- ❖Understand the behaviour of beams with openings under bending and shear
- ❖Design the opening using Mechanism Approach, Plasticity Truss Method or Strut-and-Tie Method
- ❖Detail the reinforcement to satisfy serviceability and ultimate limit states.



Beams with Openings: and Design.
Mansur, M.A. and Kiang-Hwee Tan.
CRC Press Ltd, 1999

Additional references

- Mansur, M.A., Tan, K.H. and Lee, S.L., "**A Design Method for Reinforced Concrete Beams with Large Openings**", Journal of the American Concrete Institute, Proceedings, Vol. 82, No. 4, USA, July/August 1985, pp. 517-524.
- Tan, K.H. and Mansur, M.A., "**Design Procedure for Reinforced Concrete Beams with Web Openings**", ACI Structural Journal, Vol. 93, No. 4, USA, July-August 1996, pp. 404-411.
- Mansur, M.A., Huang, L.M., Tan, K.H. and Lee, S.L., "**Analysis and Design of R/C Beams with Large Web Openings**", Journal of the Institution of Engineers, Singapore, Vol. 31, No. 4, July/August 1991, pp. 59-67.


Typical layout
Alternative layout

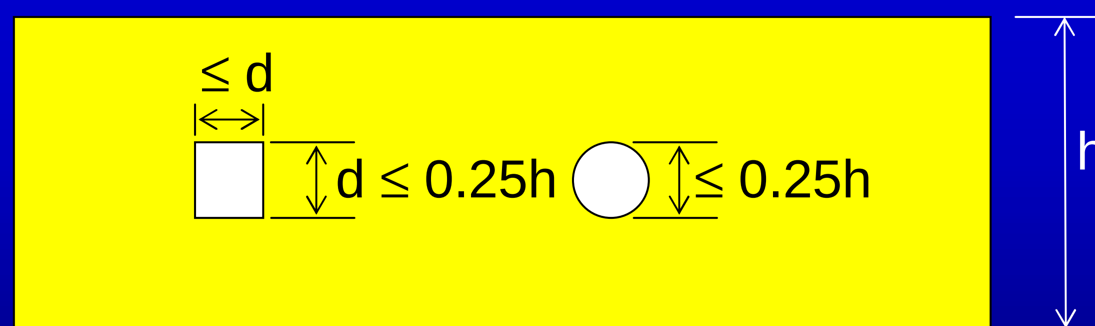
Opening in beam

Classification of Openings

- ***By geometry:***
  - **Small opening** ***if***

    Depth or diameter ≤ 0.25 x beam depth

    and Length ≤ depth

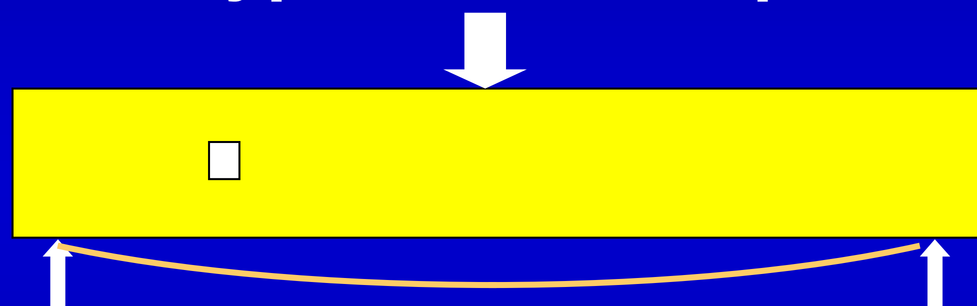


  - **Large opening** ***otherwise.***

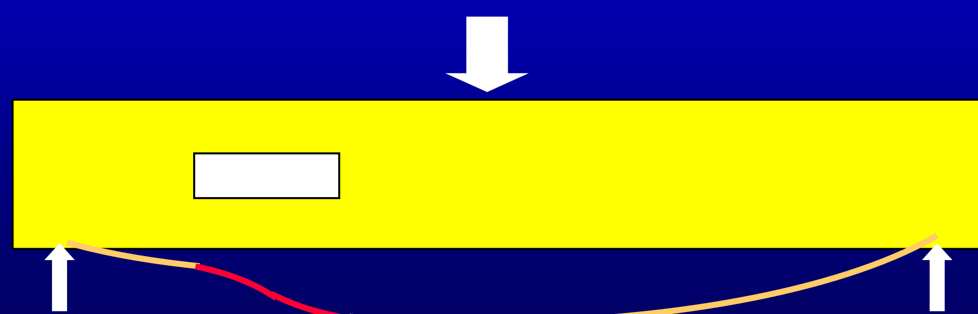
- ***By structural response:***

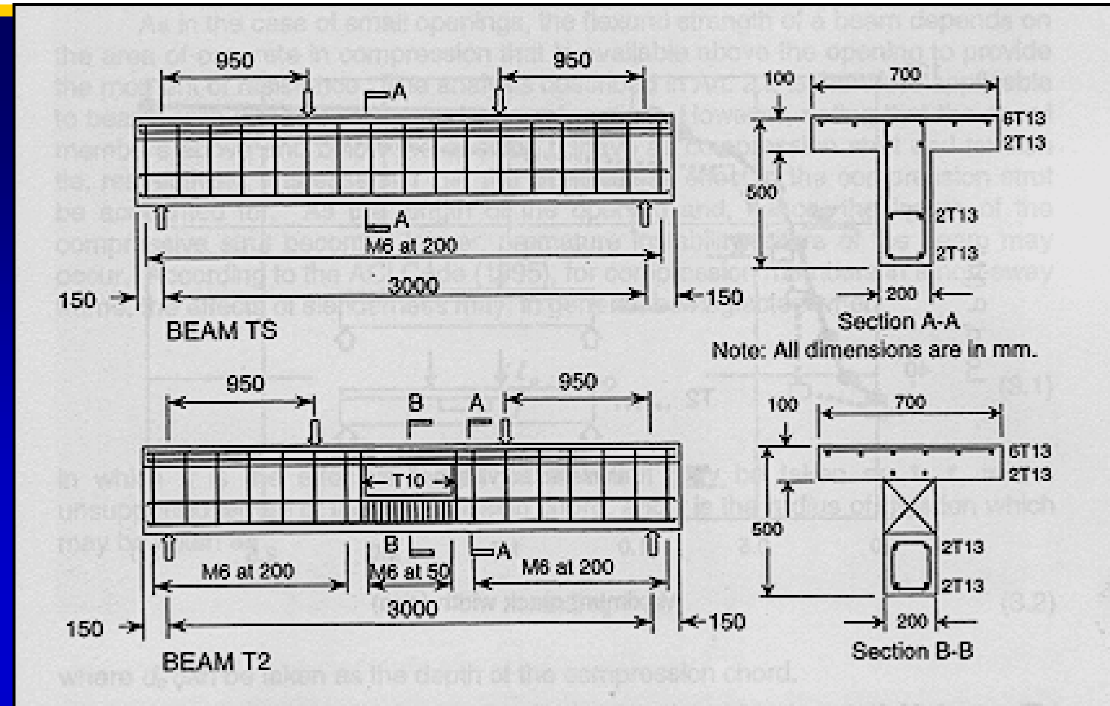
  – **<u>Small opening</u>** ***if***

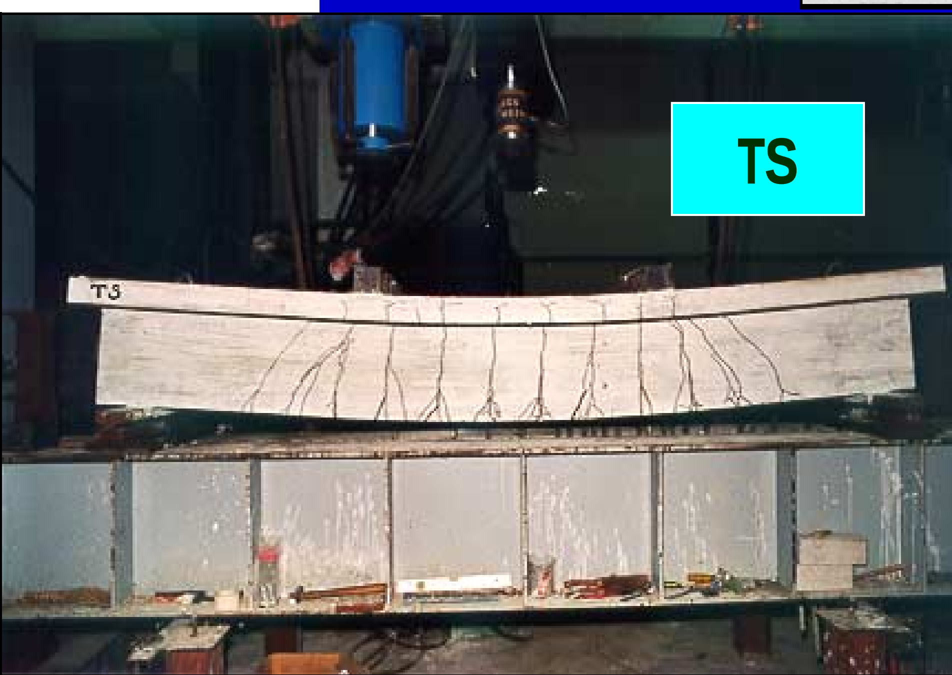
  Beam-type behaviour persists.

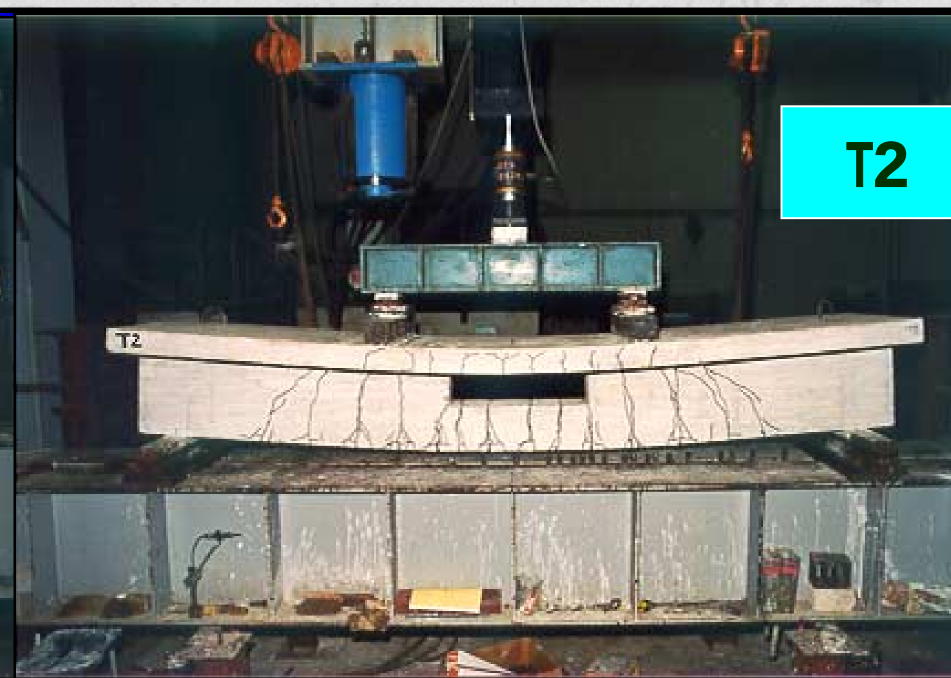


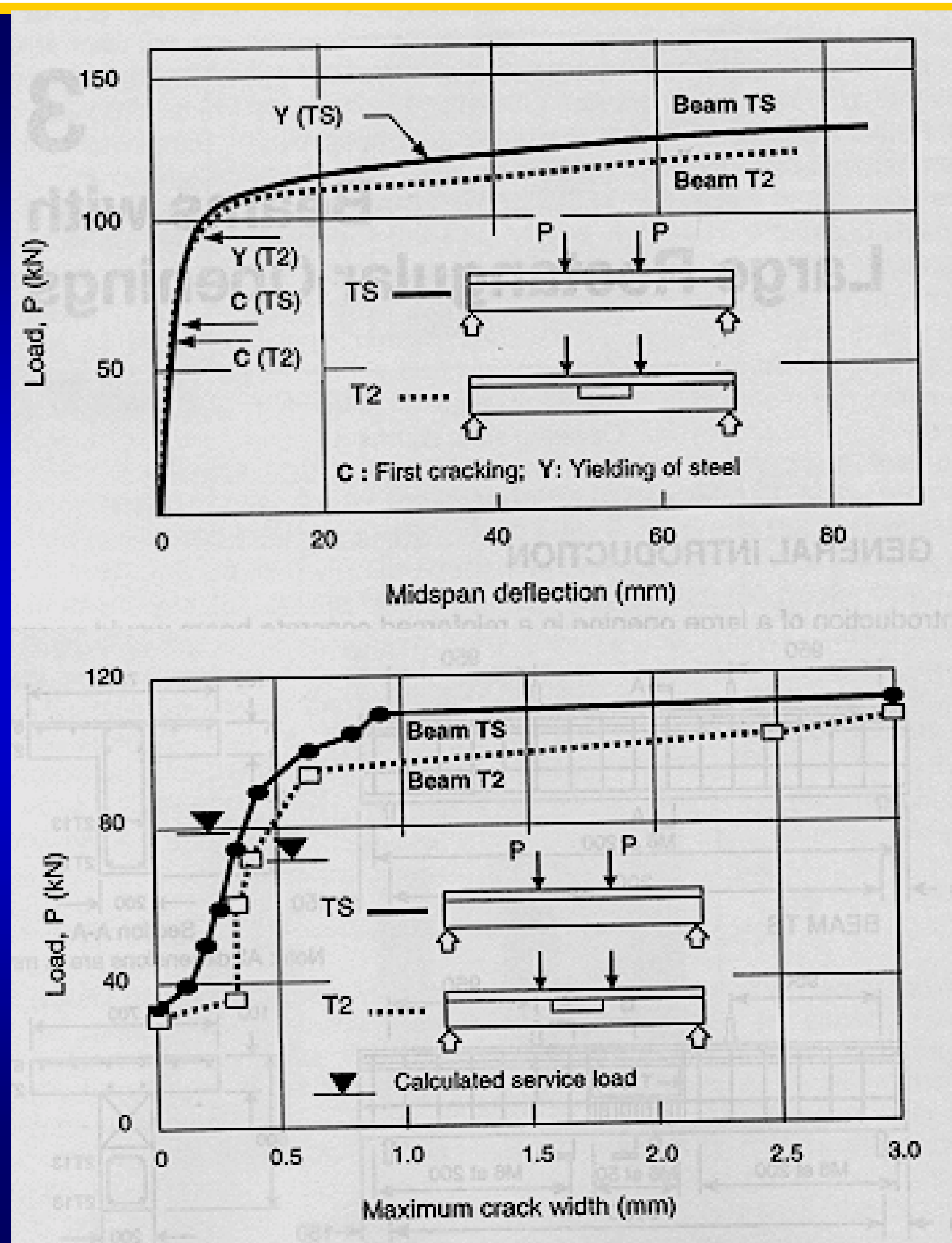
  – **<u>Large opening</u>** ***otherwise.***




950
950
A
100
700
6T13
2T13
500
2T13
2T13
A
M6 at 200
3000
150
150
200
BEAM TS
Section A-A
Note: All dimensions are in mm.
950
B
A
950
100
700
6T13
2T13
T10
500
B
A
2T13
M6 at 200
M6 at 50
M6 at 200
2T13
3000
150
150
200
BEAM T2
Section B-B


TS

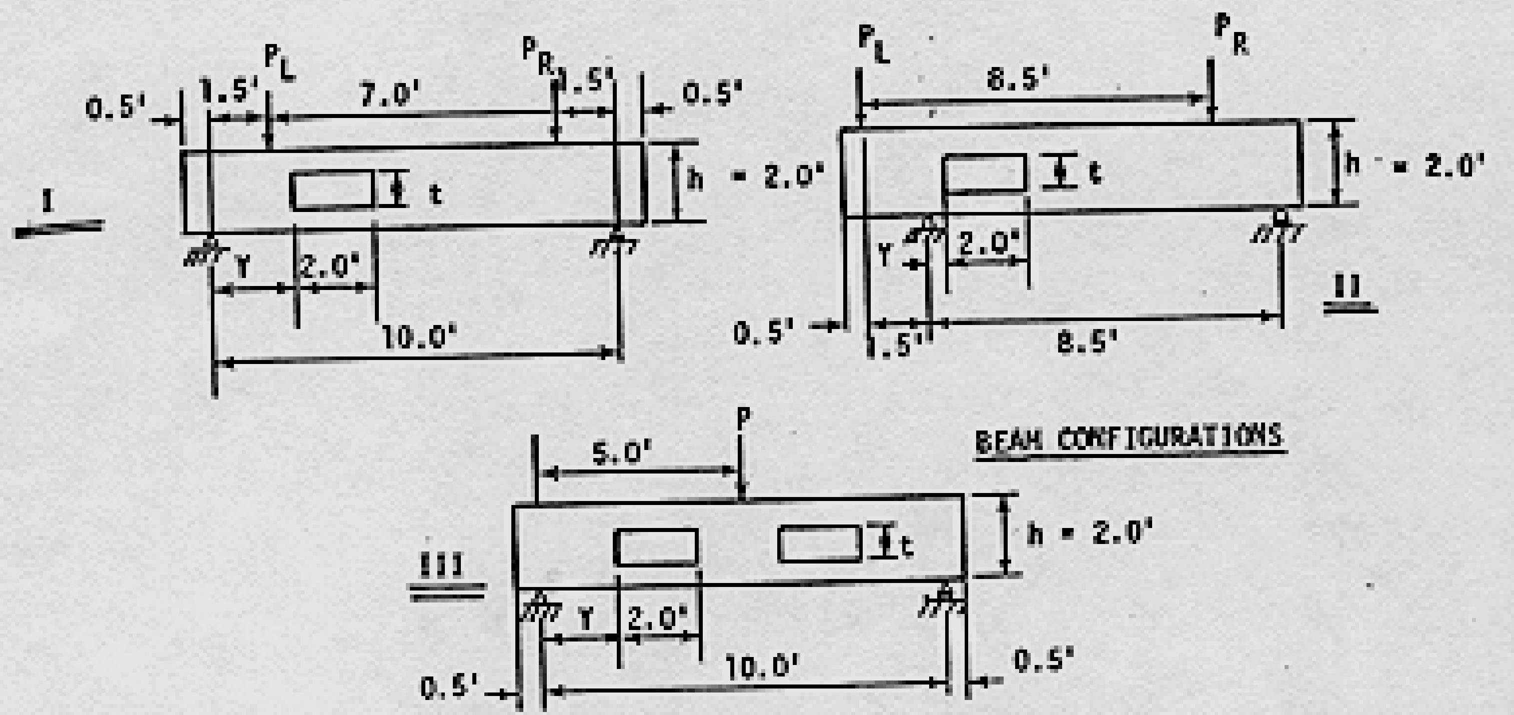

T2

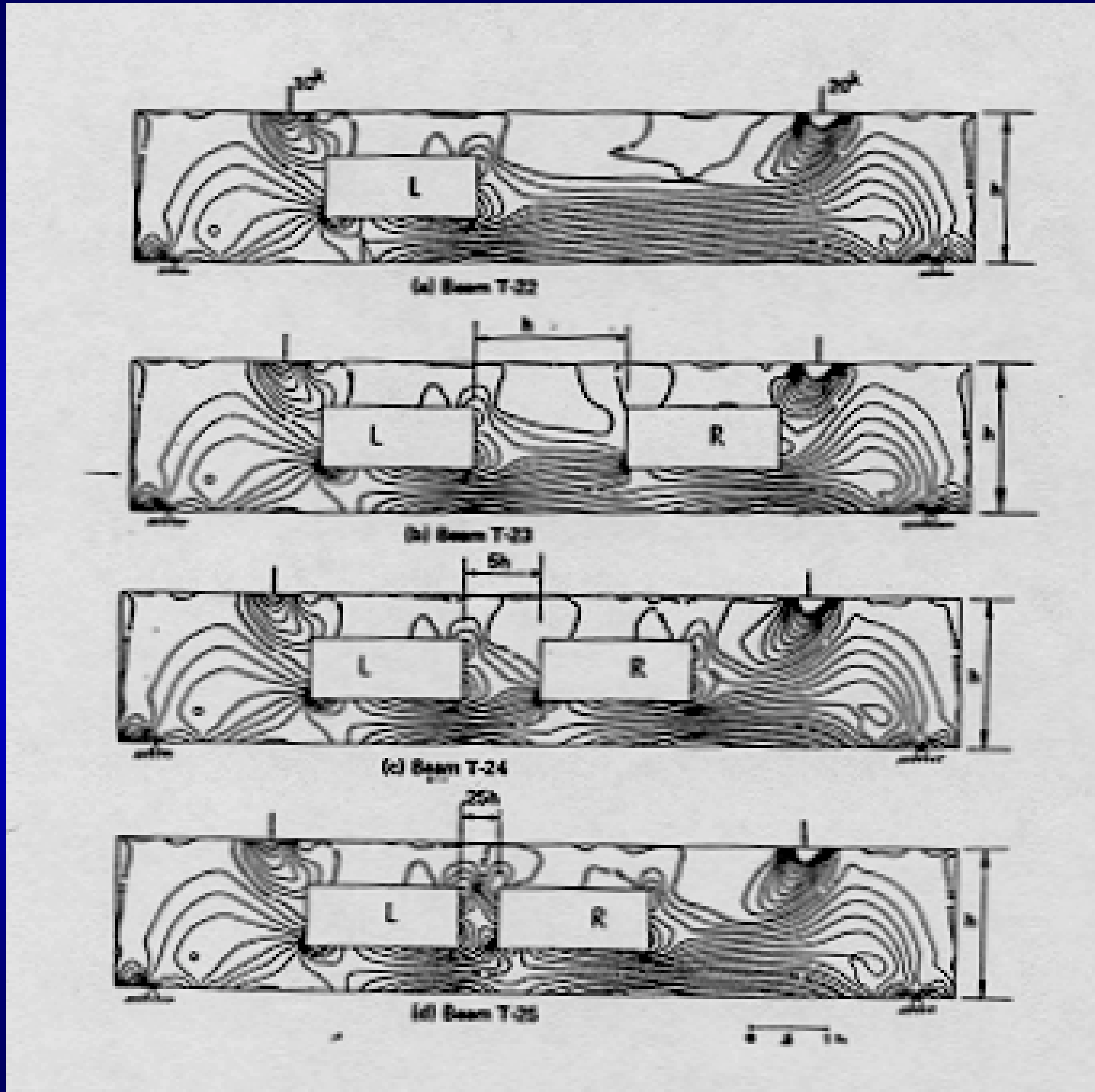

150
100
50
0
20
40
60
80
Load, P (kN)
Midspan deflection (mm)
Y (TS)
Beam TS
Beam T2
Y (T2)
C (TS)
C (T2)
P
P
TS
T2
C : First cracking; Y: Yielding of steel
120
80
40
0
0.5
1.0
1.5
2.0
2.5
3.0
Load, P (kN)
Maximum crack width (mm)
Beam TS
Beam T2
TS
T2
Calculated service load

DESIGN TOOLS

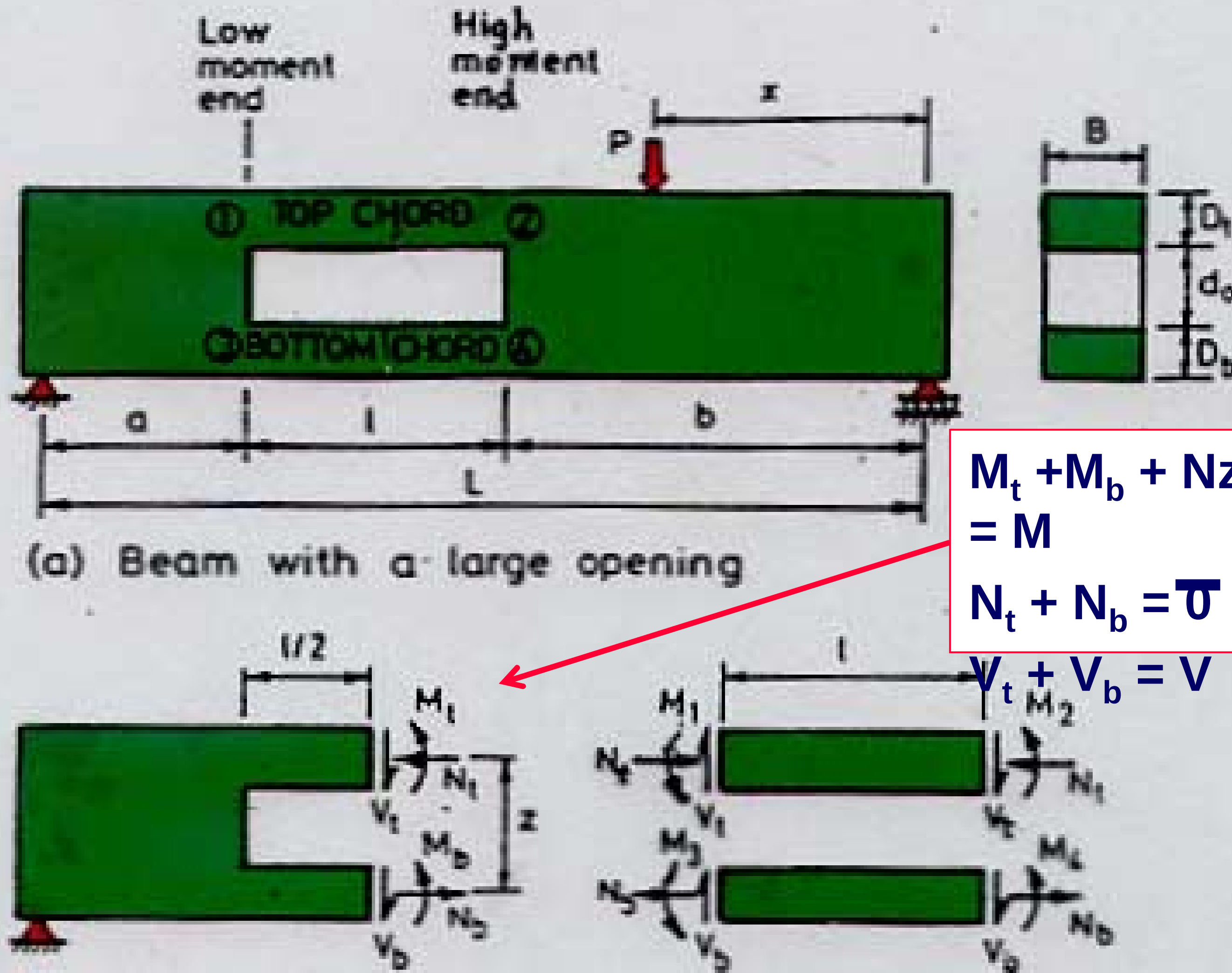
- ***Finite element analysis:***
  - **elastic stress state**
  - **area of stress concentration**
- ***Truss analysis (Strut-and-tie model):***
  - **lower bound approach**
  - **detailing for crack control and strength**
- ***Tests:***
  - **specific cases**
  - **serviceability & strength aspects**

BEHAVIOUR *of* BEAMS WITH LARGE OPENINGS





Principal tension stress contours



(a) Beam with a large opening

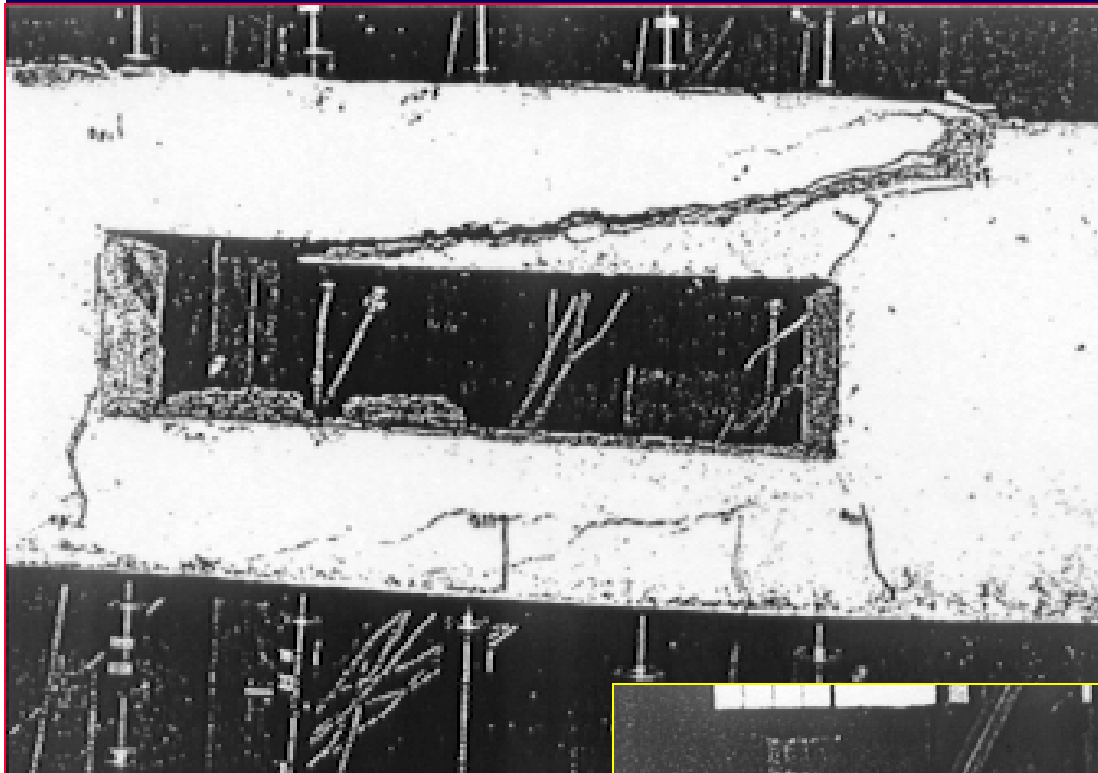
(b) Free-body diagram at opening

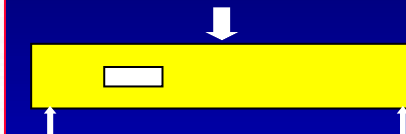
(c) Free-body diagram of chord members

$M_t + M_b + Nz = \overline{M} = M$

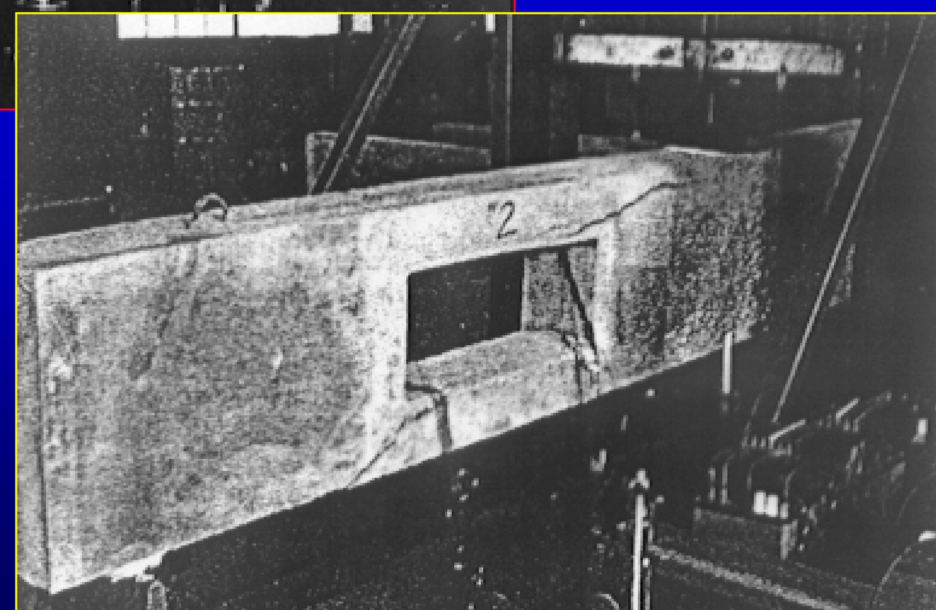
$N_t + N_b = \overline{0}$

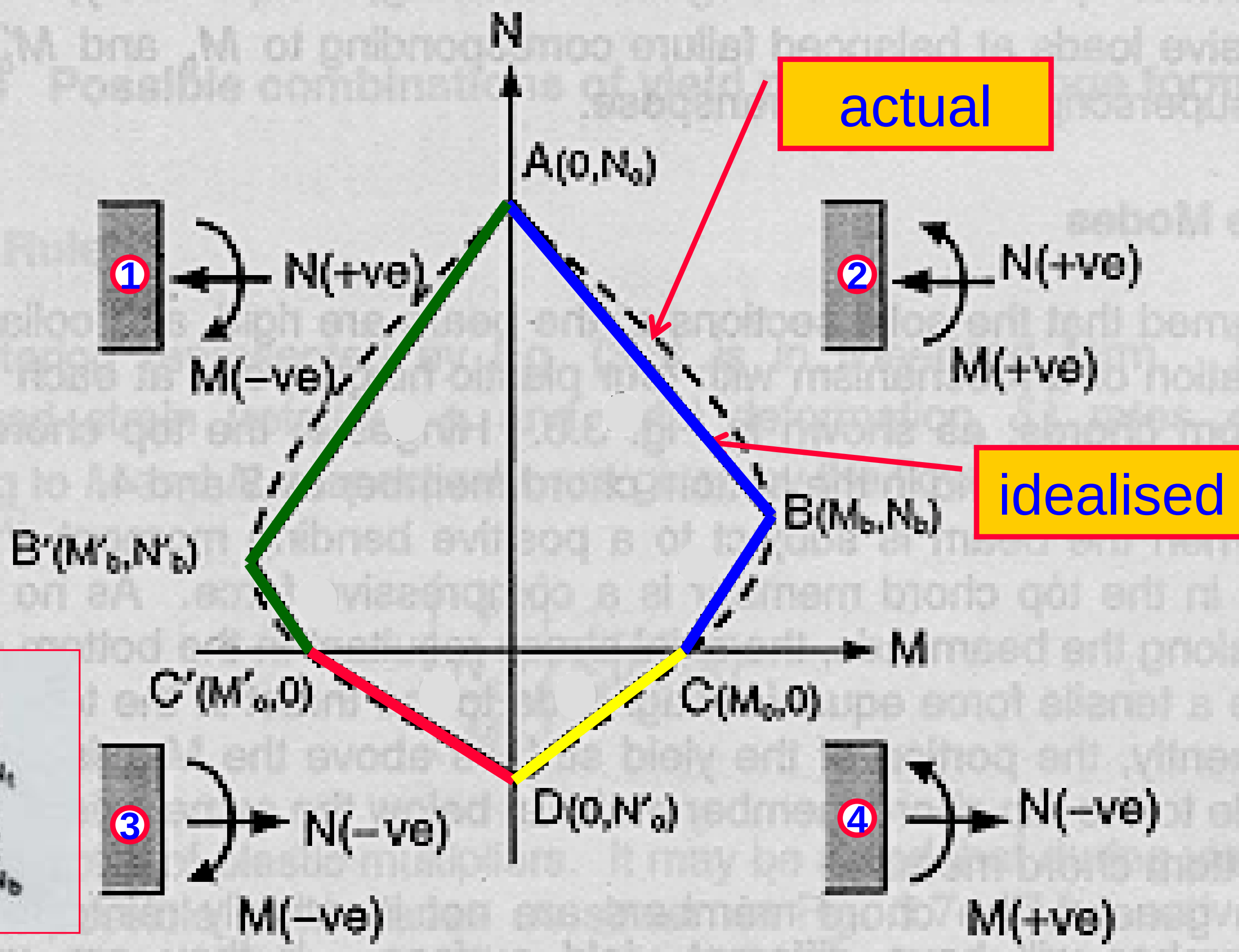
$V_t + V_b = V$

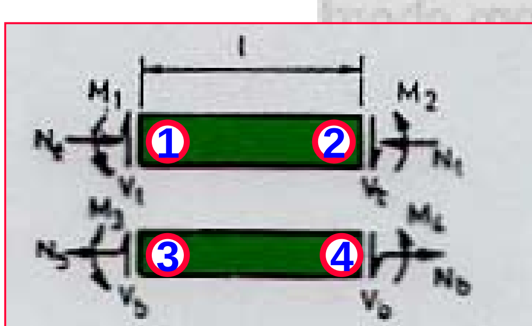


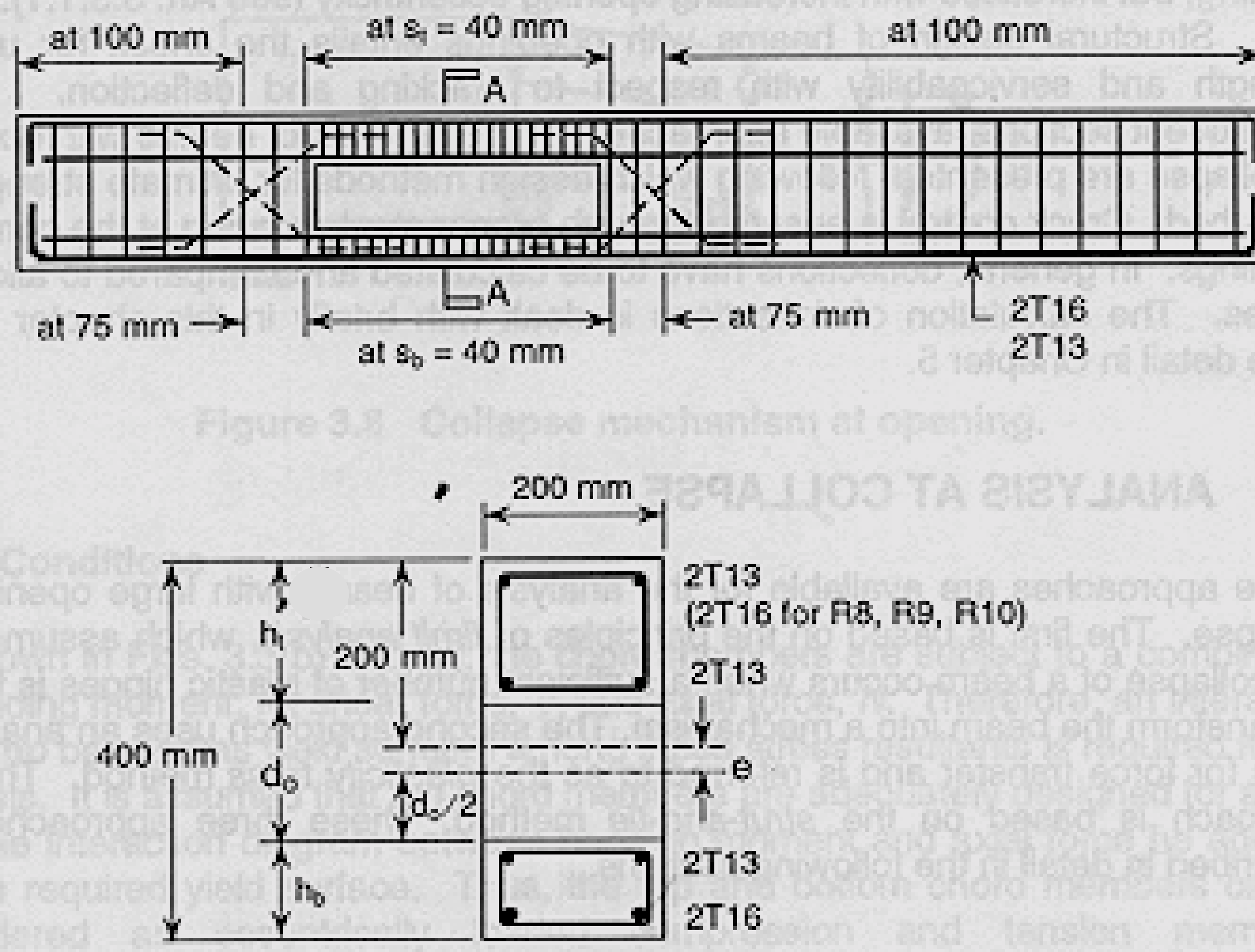


Inadequately reinforced openings

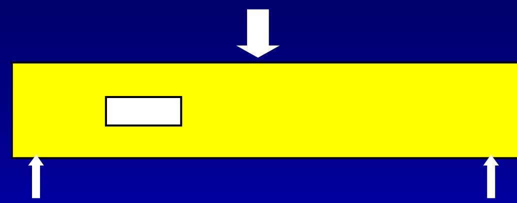


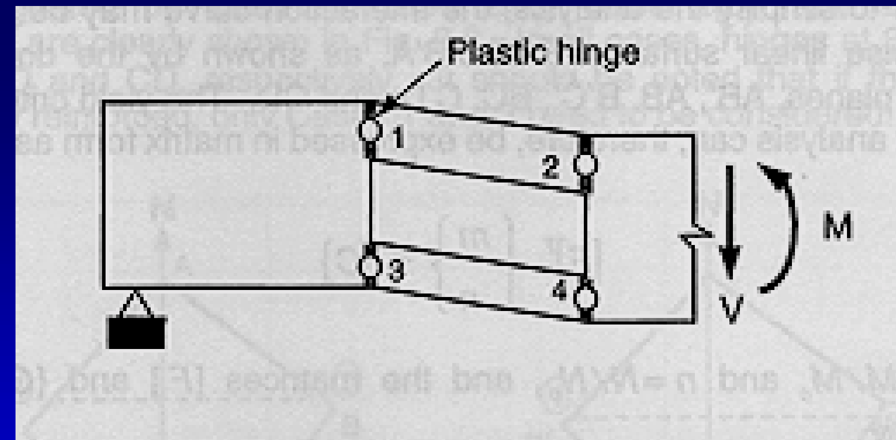

N
A(0,N₀)
actual
idealised
B(Mb,Nb)
B'(M'b,N'b)
C(M₀,0)
C'(M'₀,0)
D(0,N'₀)
M
1
N(+ve)
M(–ve)
2
N(+ve)
M(+ve)
3
N(–ve)
M(–ve)
4
N(–ve)
M(+ve)

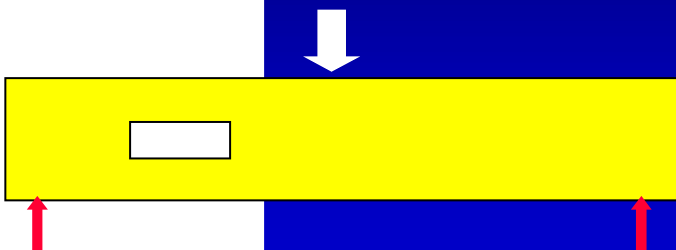

l
1
2
3
4

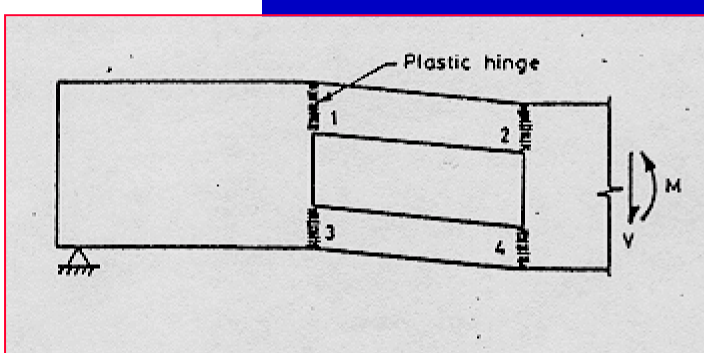

at 100 mm
at s_t = 40 mm
at 100 mm
A
A
at 75 mm
at s_b = 40 mm
at 75 mm
2T16
2T13
200 mm
2T13
(2T16 for R8, R9, R10)
2T13
h_t
200 mm
400 mm
d_o
d_o/2
e
h_b
2T13
2T16

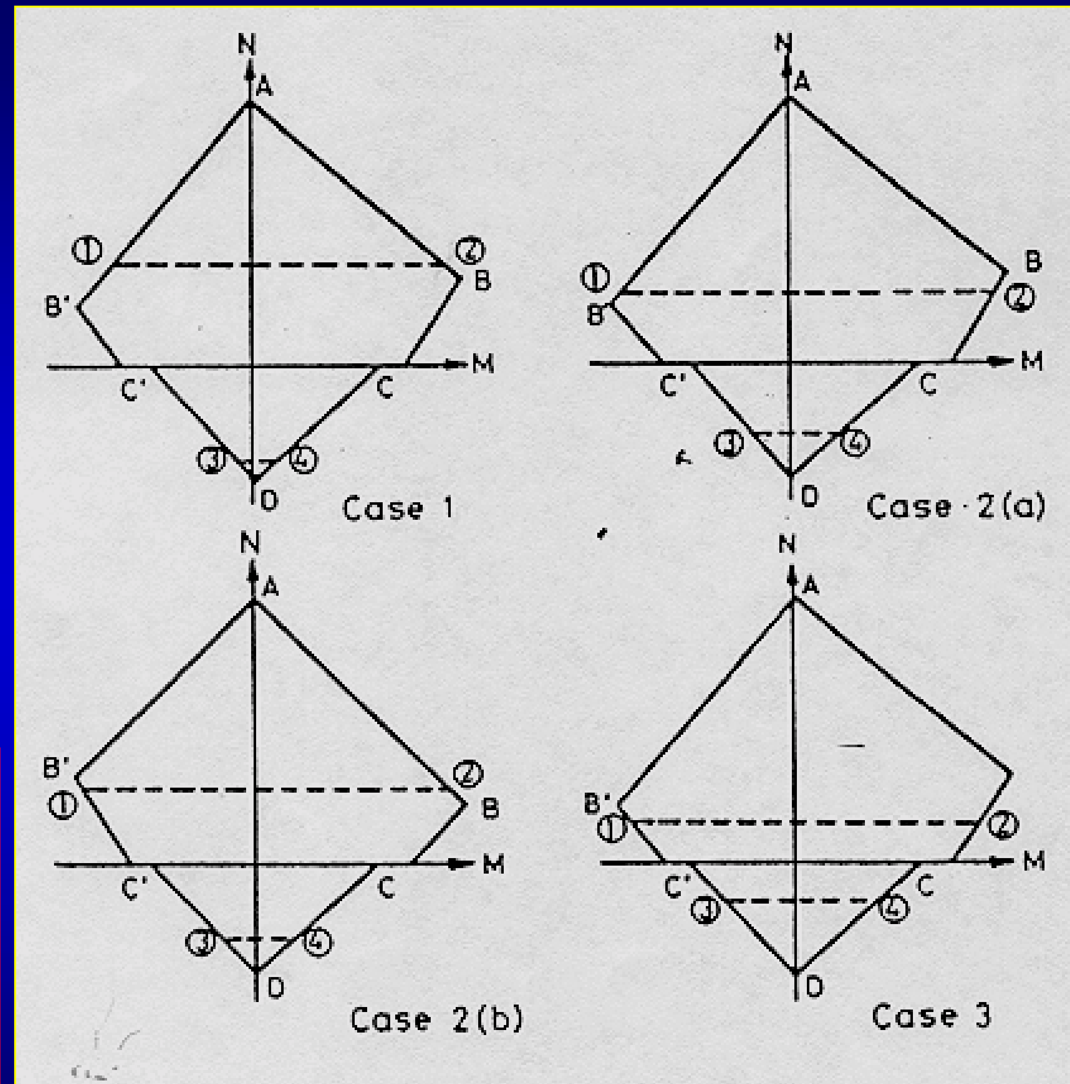
Section A-A

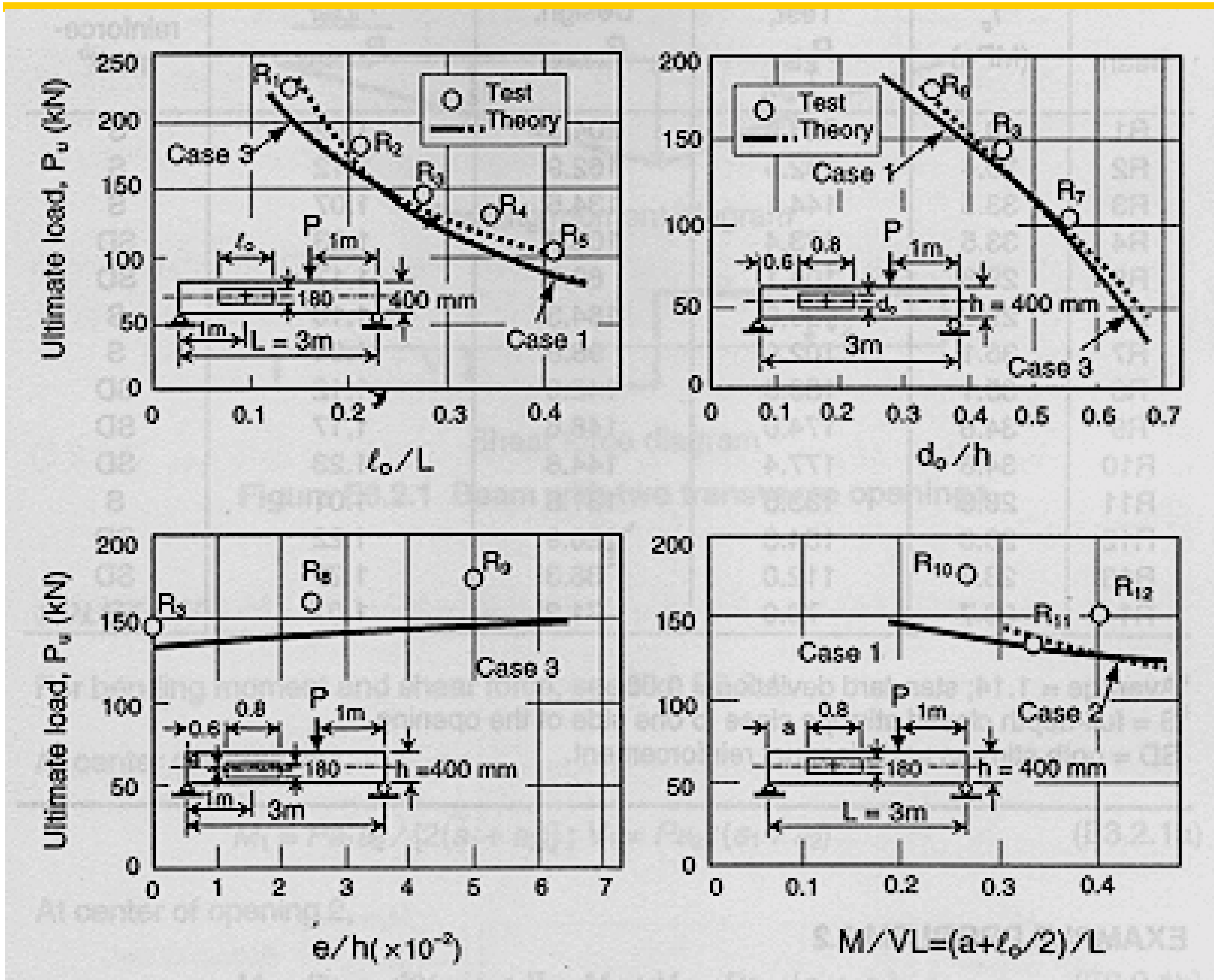


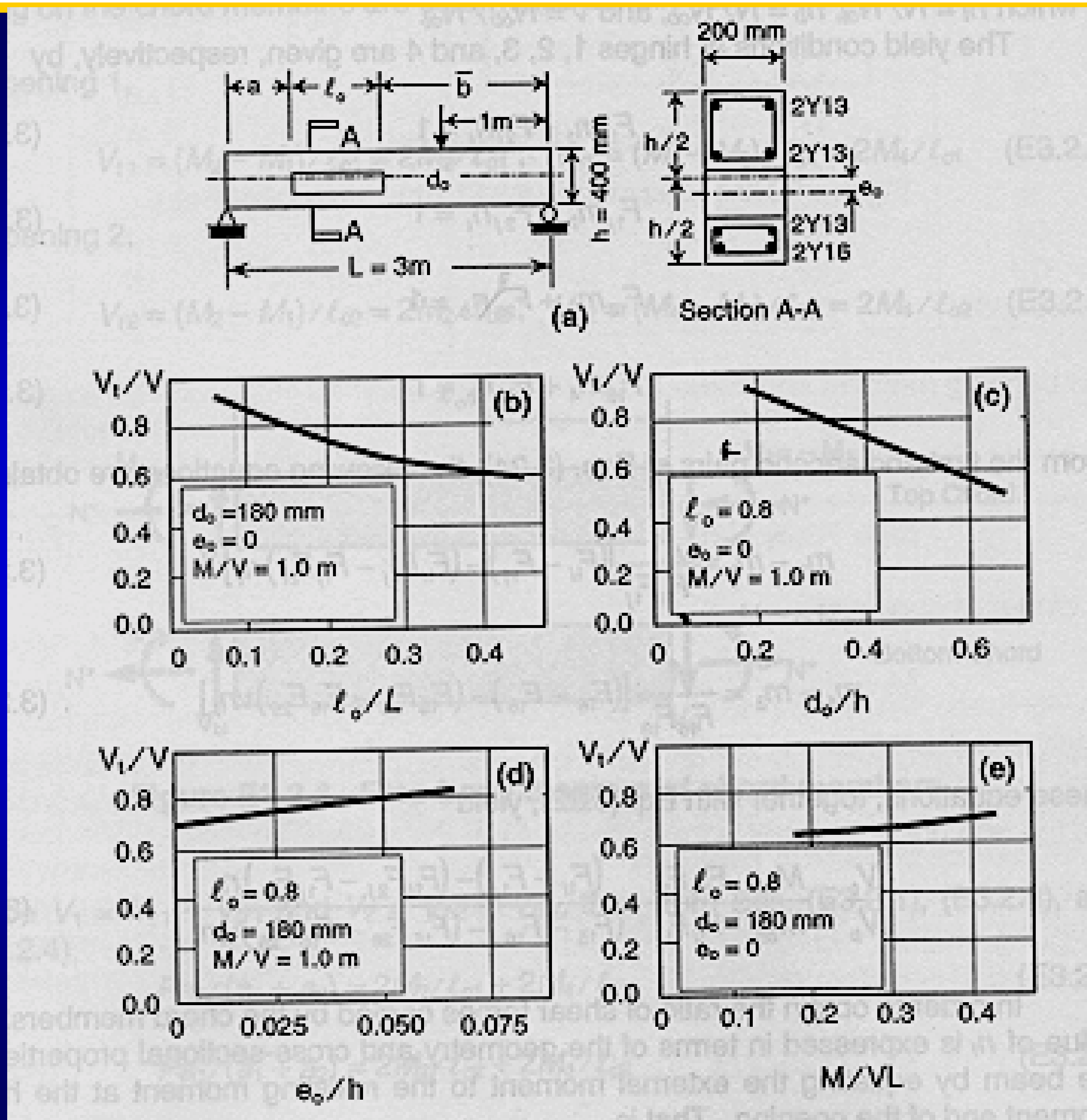

Plastic hinge
1
2
3
4
M
V

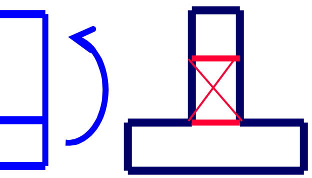


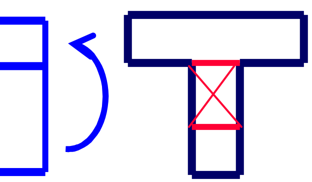

Plastic hinge
1
2
3
4
M
V

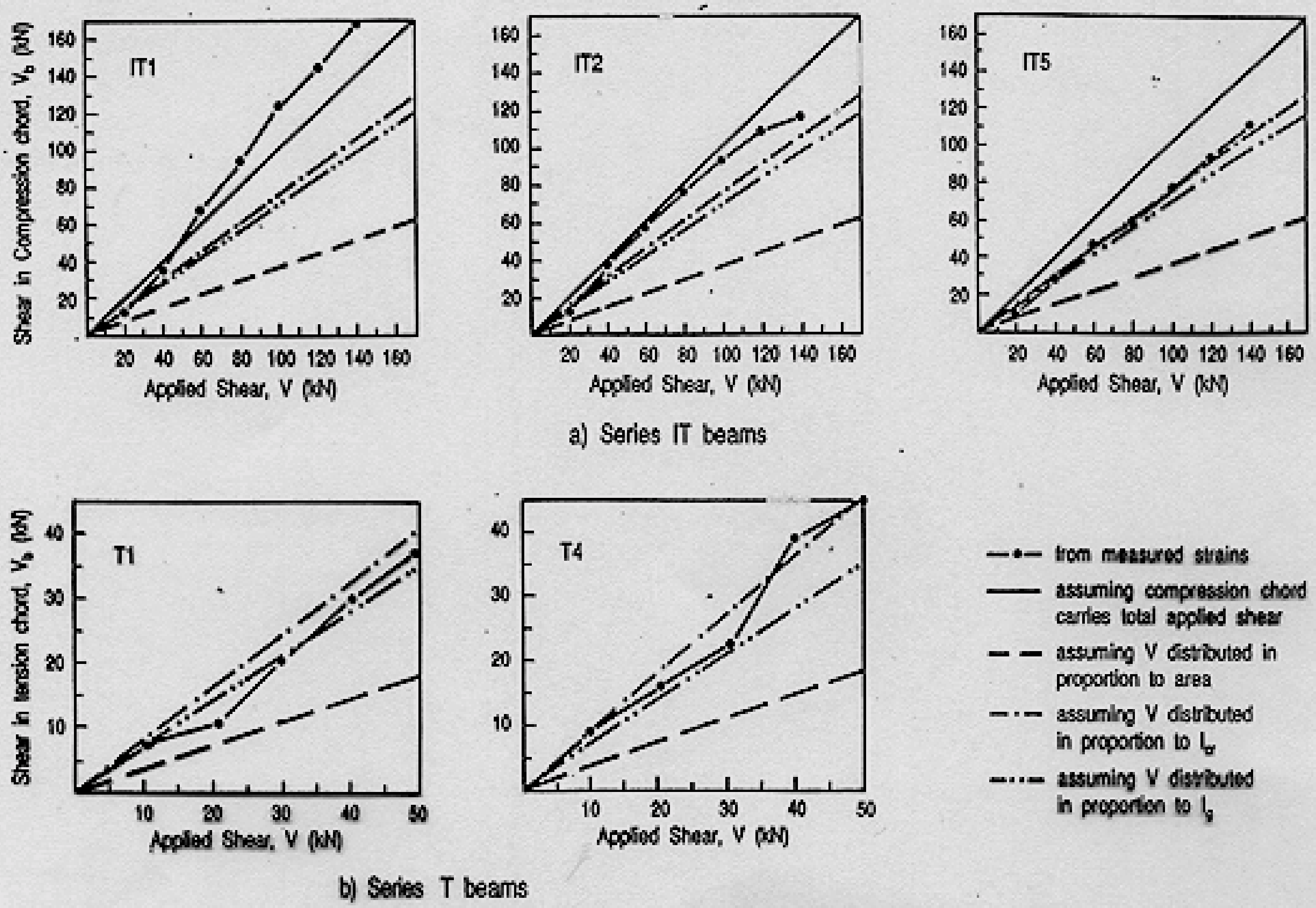

N
A
B'
B
C'
C
D
M
Case 1
Case 2(a)
Case 2(b)
Case 3


Ultimate load, Pu (kN)
Test
Theory
Case 3
Case 1
R1
R2
R3
R4
R5
P
1m
L = 3m
180
400 mm
ℓo/L
R6
R3
R7
0.6
0.8
do
h = 400 mm
3m
do/h
R8
R9
h =400 mm
e/h(×10^-2)
R10
R11
R12
Case 2
a
L = 3m
M/VL=(a+ℓo/2)/L

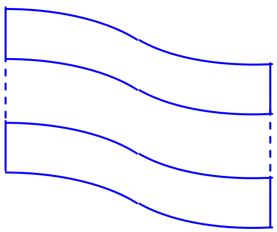

200 mm
a
ℓ_o
b
1m
A
A
d_o
h = 400 mm
L = 3m
(a)
h/2
h/2
2Y13
2Y13
2Y13
2Y16
e_o
Section A-A
(b)
d_o =180 mm
e_o = 0
M/V = 1.0 m
V_t/V
ℓ_o/L
(c)
ℓ_o = 0.8
e_o = 0
M/V = 1.0 m
d_o/h
(d)
ℓ_o = 0.8
d_o = 180 mm
M/V = 1.0 m
e_o/h
(e)
ℓ_o = 0.8
d_o = 180 mm
e_o = 0
M/VL

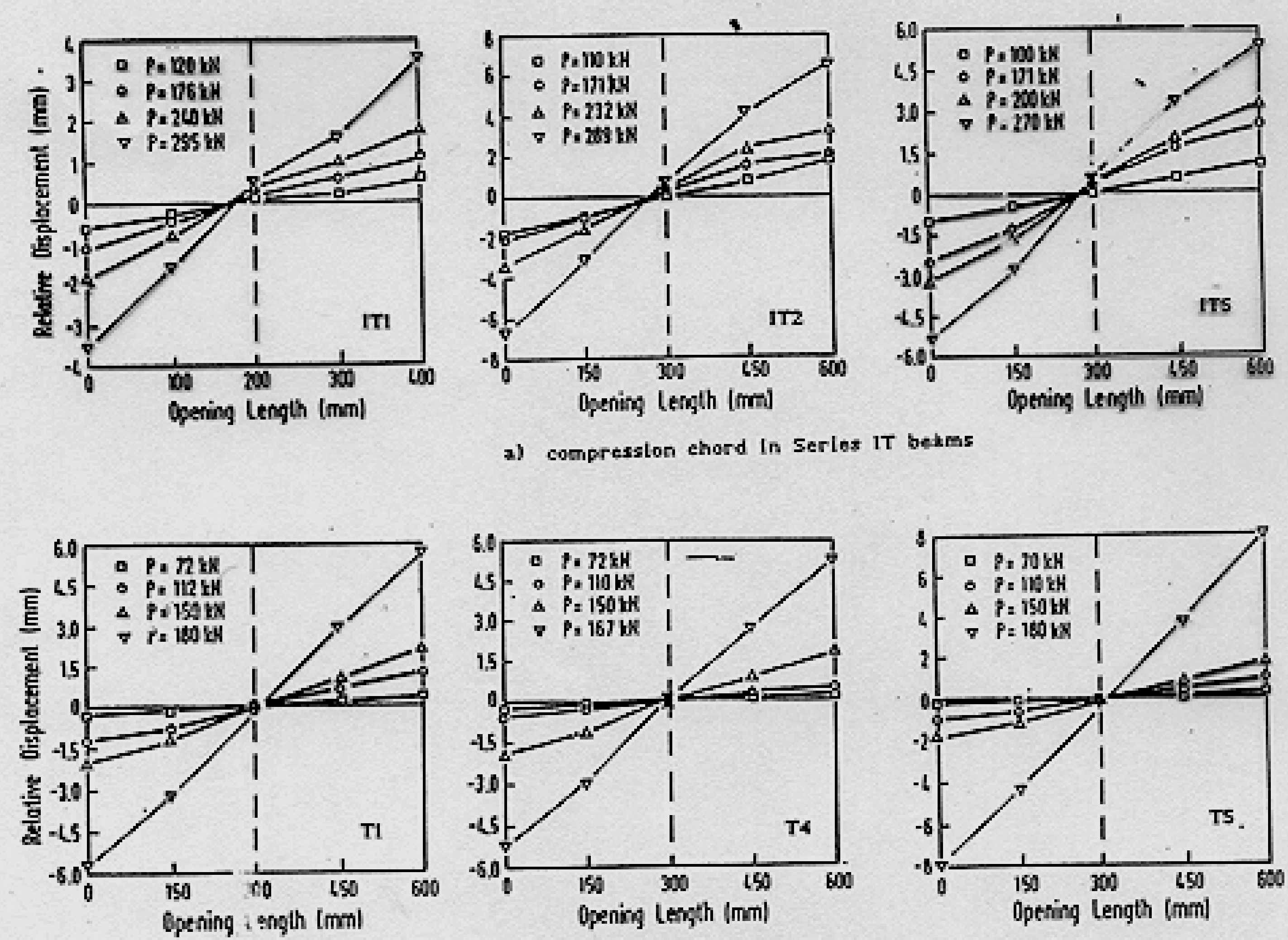






Shear carried by chords





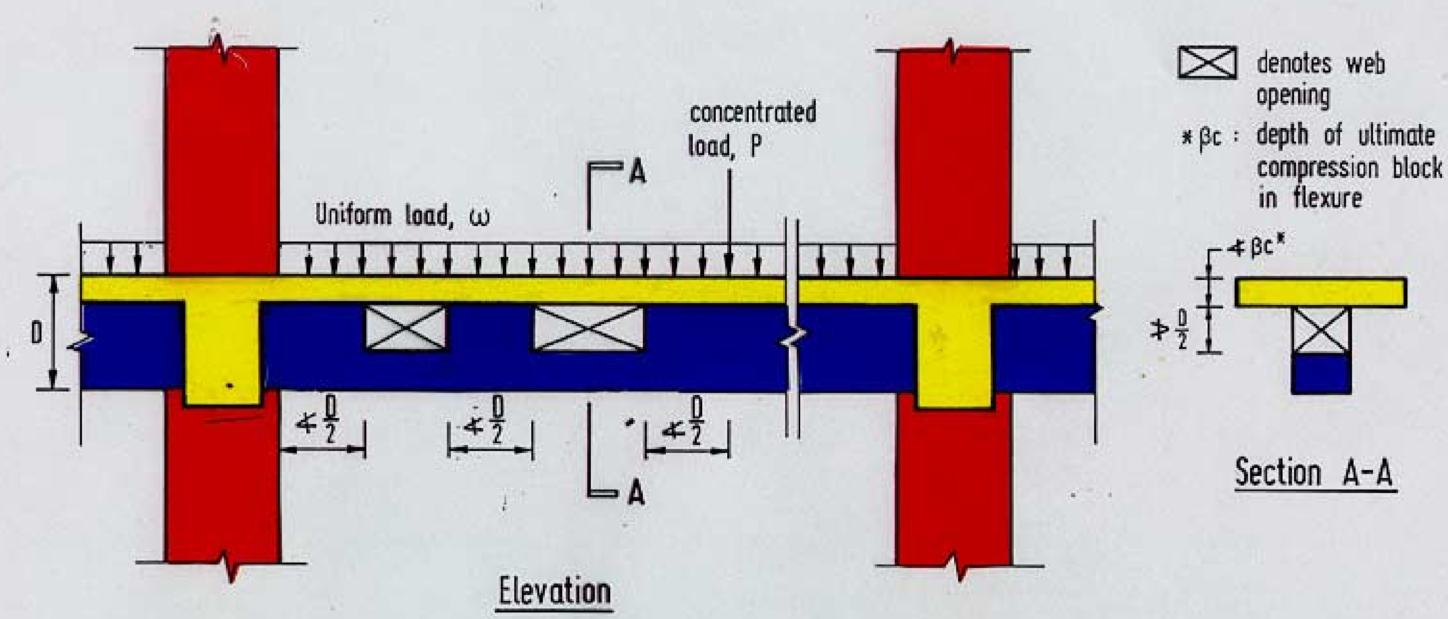
a) compression chord in Series IT beams

b) tension chord in Series T beams

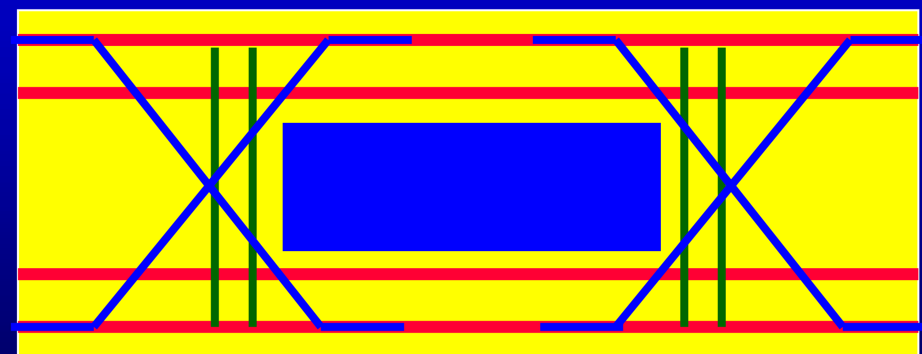
Contra-flexural points

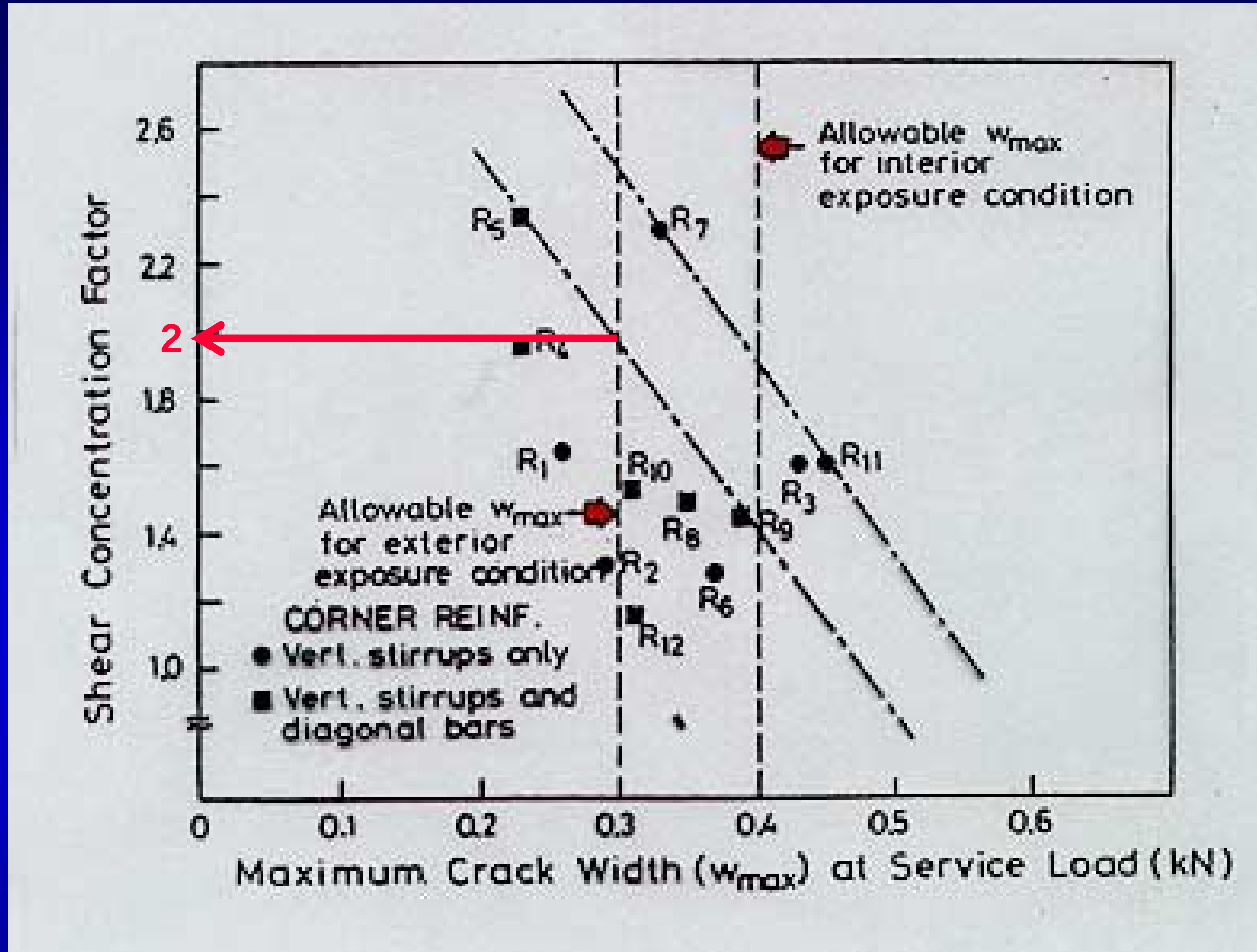
DESIGN & DETAILING *of* OPENINGS - *General guidelines*

- sufficient area to develop ultimate compression block in flexure
- openings to be located more than D/2 from supports, concentrated loads and adjacent openings
- opening depth ≤ D/2
- opening length
  - stability of chord member
  - deflection requirement of beam


denotes web opening
* βc : depth of ultimate compression block in flexure
concentrated load, P
Uniform load, ω
A
A
D
≮ D/2
≮ D/2
≮ D/2
≮ βc*
≯ D/2
Section A-A
Elevation

- ***Crack control***
  - **principal tensile stress direction**
  - **stress concentration**
  - **congestion of reinforcement**





⇒shear concentration factor of 2 (i.e. $2V_u$)

⇒diagonal bars to carry 50~75% of total shear; vertical stirrups to carry the rest

- **Strength requirement**
  - **2 general approaches:**
    - ***mechanism approach***
      - ◊ **failure at an opening is due to the formation of four plastic hinges, one at each corner of the opening**
    - ***truss model approach***
      - ◊ **plasticity truss method**
      - ◊ **strut-and-tie method**
      - ◊ **applied loads are transferred across opening to supports by a truss system**
      - ◊ **lower bound approaches**

Mechanism Approach

- **Rigorous method**

1. Calculate M_u and V_u at centre of opening.

2. Assume suitable reinft. for chord members.

3. Determine N_u in chord members and hinge moments $(M_u)_{*,*}$ from

$$N_u = [M_u - (M_s)_t - (M_s)_b]/z$$

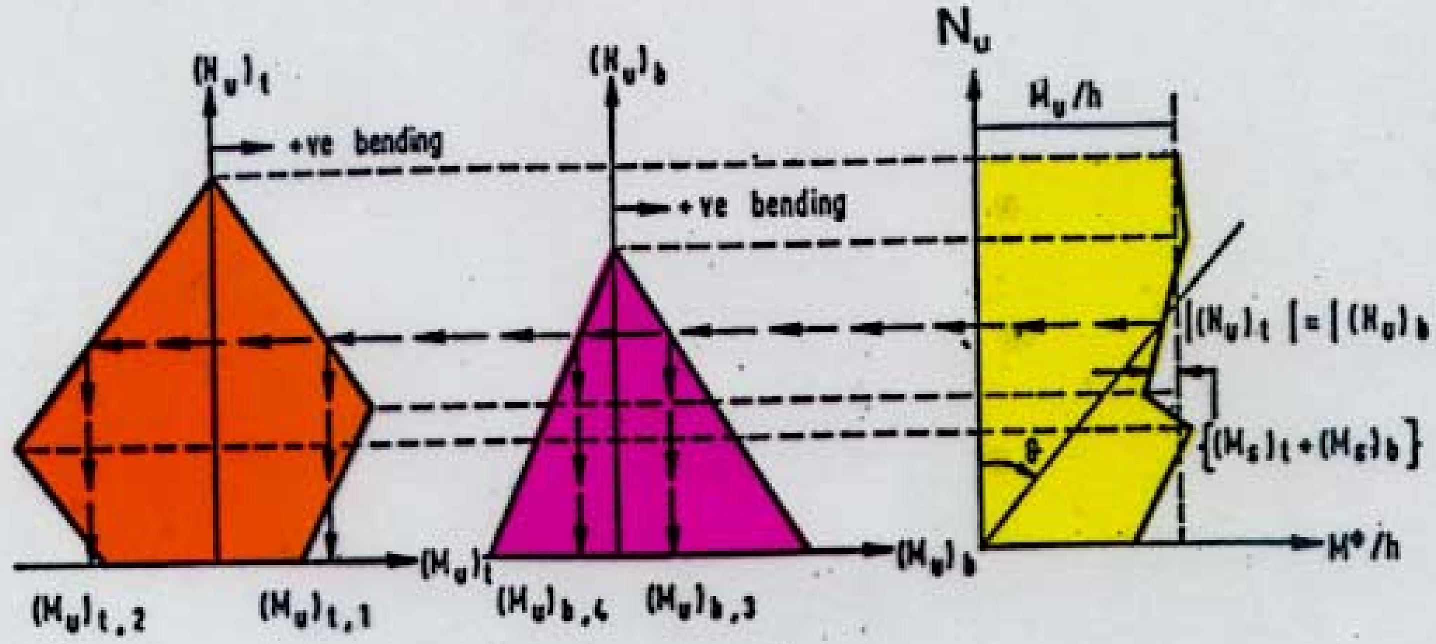
Slide 15

where

$$(M_s)_t = [(M_u)_{t,1} + (M_u)_{t,2}]/2$$

$$(M_s)_b = [(M_u)_{b,4} + (M_u)_{b,3}]/2$$

and $(M_u)_{*,*}$ are related to N_u by the respective M-N interaction diagrams.



(a) Interaction diagram for top (compression) chord

(b) Interaction diagram for bottom (tension) chord

(c) Graph of N_u Vs M^*/h ($\theta = \tan^{-1} \frac{z}{h}$)

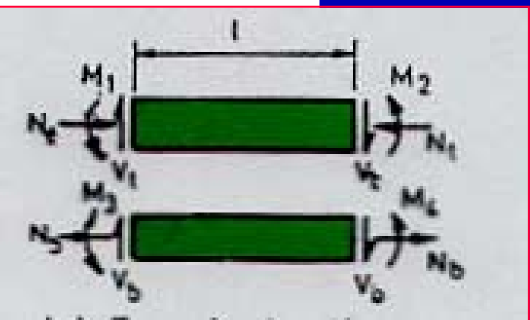
$$N_u \frac{z}{h} = \{M_u - \frac{1}{2}[(M_u)_{t,1} + (M_u)_{t,2} + (M_u)_{b,3} + (M_u)_{b,4}]/h\} = \frac{M^*}{h}$$

$$\theta = \tan^{-1}(\frac{z}{h}) = \tan^{-1}(\frac{M^*}{N_u h})$$

4. Calculate V_u in chord members from

$$(V_u)_t = [(M_u)_{t,2} - (M_u)_{t,1}]/\ell$$

$$(V_u)_b = [(M_u)_{b,4} - (M_u)_{b,3}]/\ell$$



where ℓ is the opening length.

5. Check that $(V_u)_t + (V_u)_b \cong V_u$.

- **Simplified method**
  - Unknowns: N_t, N_b, M_t, M_b, V_t, V_b
  - 3 equations:

$$M_t + M_b + Nz = M$$

$$N_t + N_b = 0$$

$$V_t + V_b = V$$

$\Rightarrow$ Assume

$$M_t = 0$$

$$M_b = 0$$

$$V_b = kV_t$$

where

$$k = 0$$

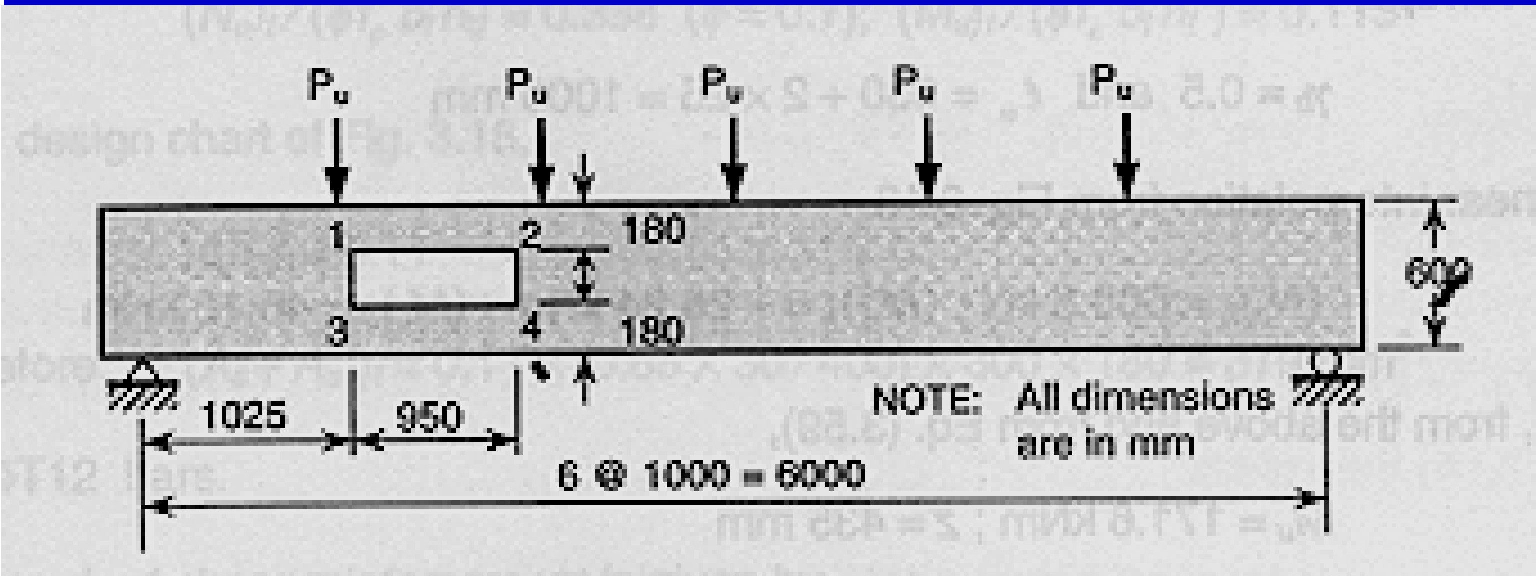
$$k = A_b/A_t$$

or

$$k = I_b/I_t$$

EXAMPLE

A simply supported reinforced concrete beam, 300 mm wide and 600 mm deep, contains a rectangular opening and is subjected to a series of point loads as shown. Design and detail the reinforcement for the opening if the design ultimate load P_u is 52.8 kN. Material properties are: f_c'=30 MPa, f_{yv}=250 MPa, and f_y=460 MPa.



Solution by Mechanism Approach

Assume point of contraflexure at midpoint of chord members and that 40% of shear is carried by bottom chord member.

At centre of opening,

M_u = 171.6 kNm

V_u = 79.2 kN

1. Bottom chord member

$(N_u)_b = M_u/z = 408.6$ kN
$(V_u)_b = 0.4\ V_u = 31.7$ kN
$(M_u)_b = (V_u)_b\ell/2 = 15.1$ kNm

$2A_s = 1954\ mm^2$
(3T20 top + 4T20 bottom)

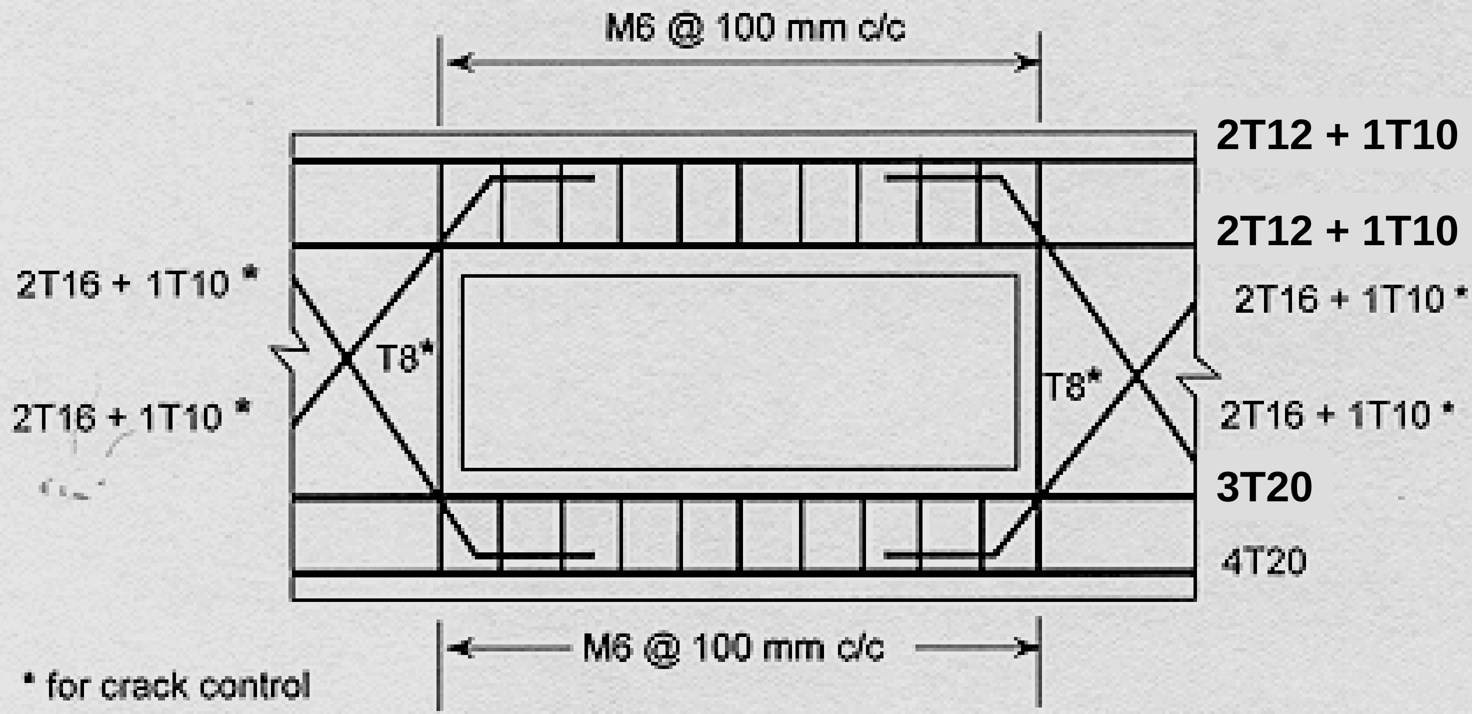
$A_{sv}/s = 400\ mm^2/m$ *(minimum)*
(M6 @ 100 mm spacing)

2. Top chord member

$(N_u)_t$=408.6 kN; $(V_u)_t$=47.5 kN; $(M_u)_t$=22.6 kNm

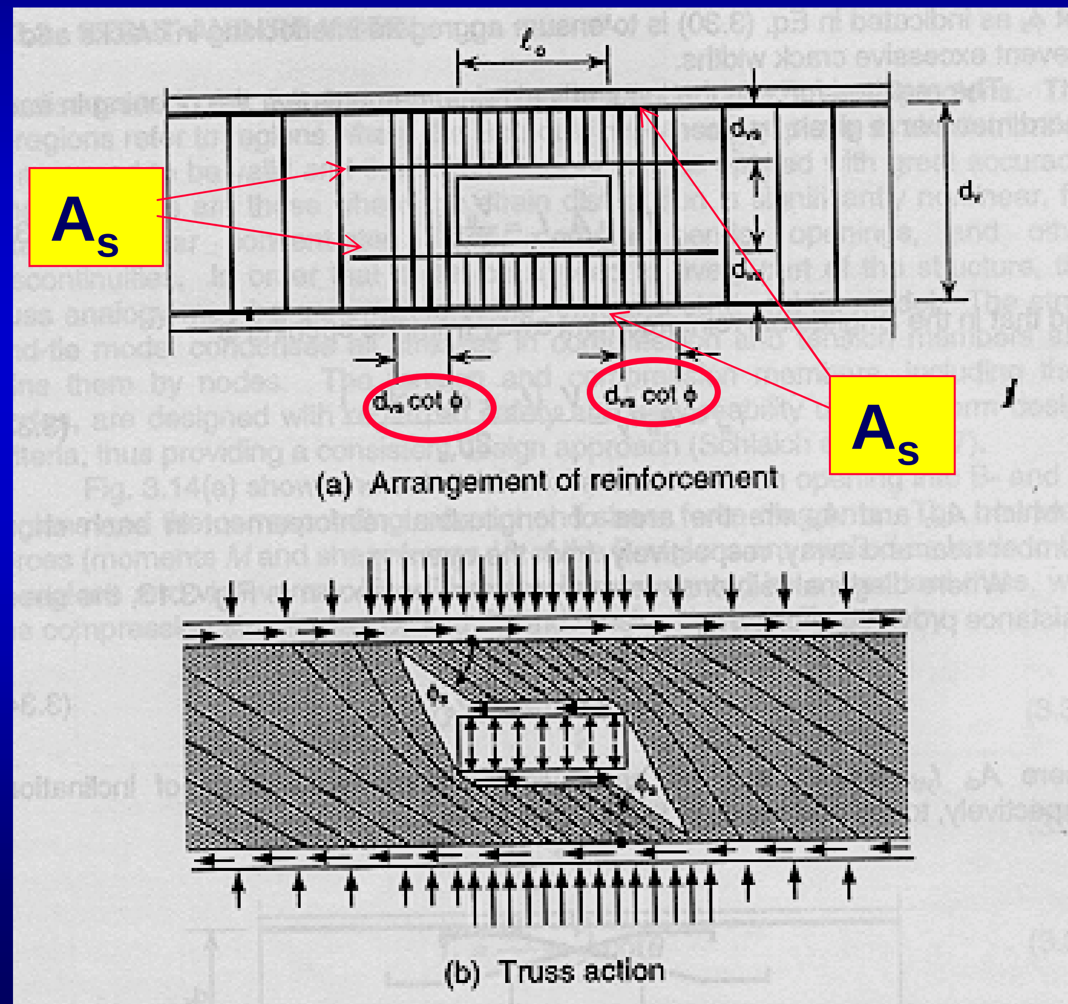
$2A_s$ = 516 mm^2
(2T12 +1T10 top & bottom)

A_{sv}/s = 525 mm^2/m
(M6 @ 100 mm spacing)


M6 @ 100 mm c/c
2T12 + 1T10
2T12 + 1T10
2T16 + 1T10 *
T8*
2T16 + 1T10 *
T8*
2T16 + 1T10 *
2T16 + 1T10 *
3T20
4T20
M6 @ 100 mm c/c
* for crack control

Truss Model Approaches

- **Plasticity truss** (AIJ approach)



© Tan KH
NUS

- ***Shear capacity***

$$V_u = 2\, b d_{vs} \rho_v f_{yv} \cot \phi_s$$

where

$$\cot \phi_s = \sqrt{(\nu f_c'/\rho_v f_{yv} - 1)} \le 2$$

$$\nu f_c'/\rho_v f_{yv} \ge 1$$

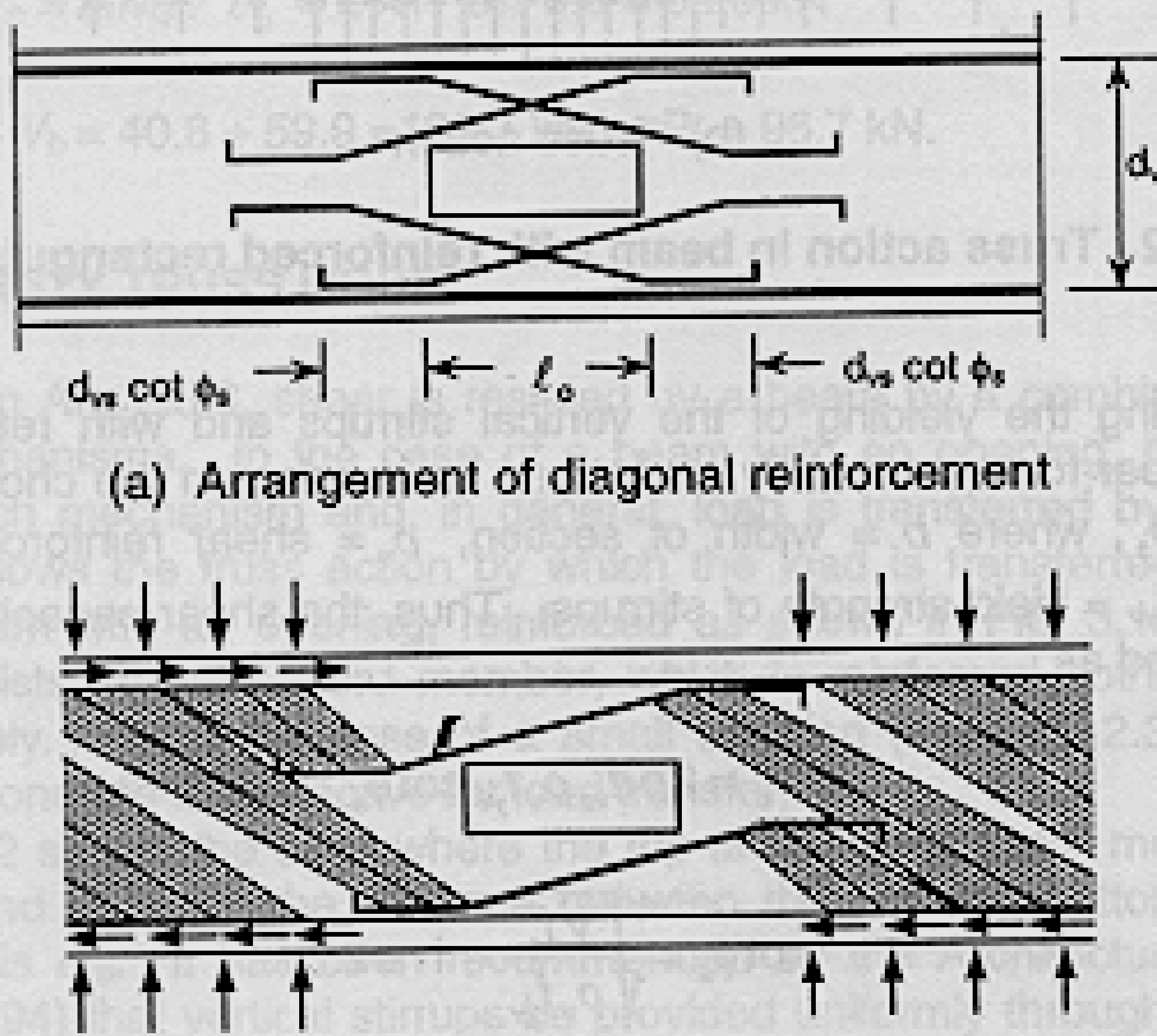
$$\rho_v = A_v f_{yv}/bs$$

$$\nu = 0.7 - f_c'/200$$

- ***Longitudinal reinforcement***

$$T_{sn} = A_{sn} f_y = V_u \ell / 2 d_{vs}$$

$$T_{sf} = A_{sf} f_y = V_u(\ell + d_{vs} \cot \phi_s)/2 d_{vs}$$



$V_d = A_{sd} f_{yd} \sin \theta_d$

(b) Truss action

Solution by Plasticity Truss Method (AIJ)

1. **Calculate v.**

$$v = V_u/(2bd_{vs}\nu f_c') = 0.089$$

2. **Determine shear reinforcement.**

From $v = \sqrt{\psi(1-\psi)}$, obtain

$$\psi \equiv \rho_v f_{yv}/\nu f_c' = 0.008$$

$$\rightarrow \rho_v = 0.00053 < \rho_{v,min} = 1/3f_y = 0.01$$

$$A_{sv}/s = 400\ mm^2/m$$

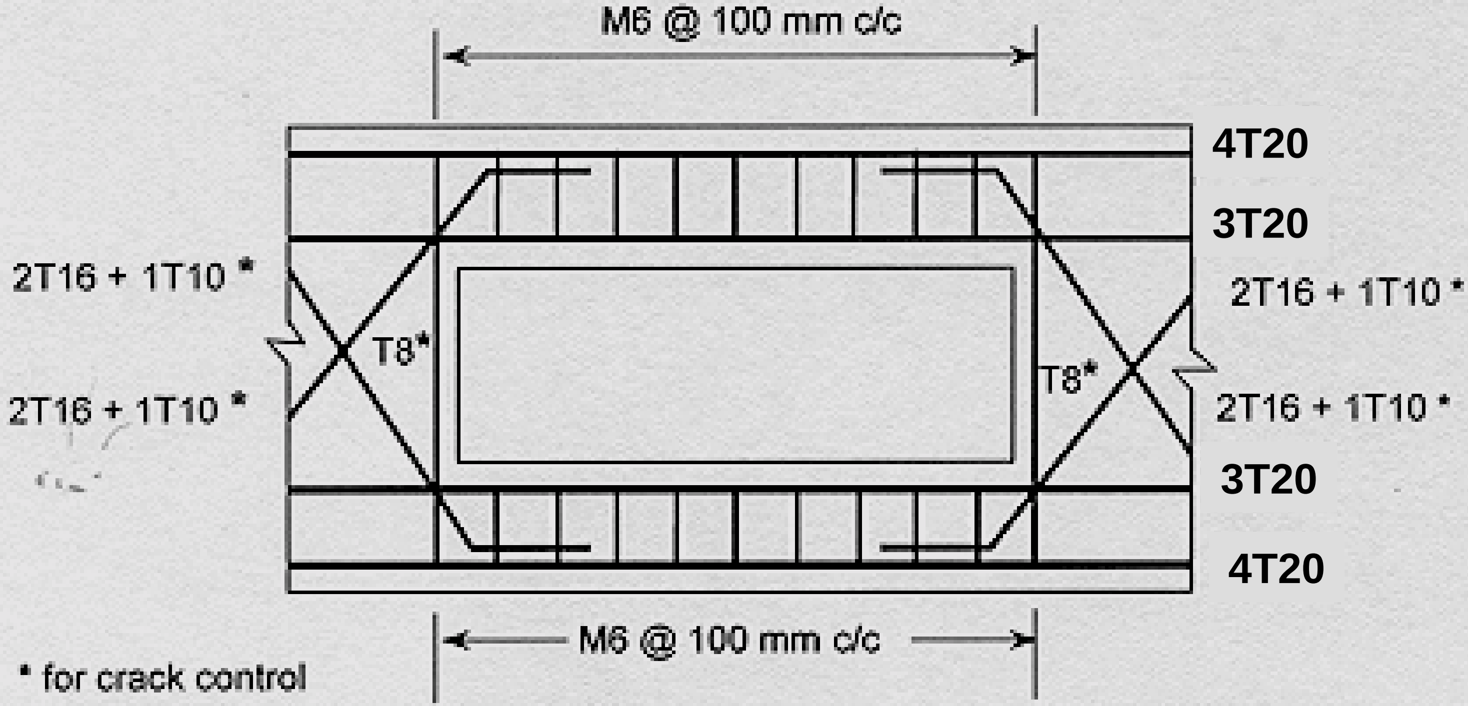
(M6 @ 100 mm spacing)

3. Determine longitudinal reinforcement.

$$A_{sn} = V_u \ell / 2 f_y d_{vs} = 957 \text{ mm}^2$$

$$A_{sf} = V_u(\ell + d_{vs} \cot \phi_s)/2 f_y d_{vs}$$
$$= 1080 \text{ mm}^2$$

$\Rightarrow$ 3T20 (near) + 4T20 (far)



Strut-and-tie method

General Principles

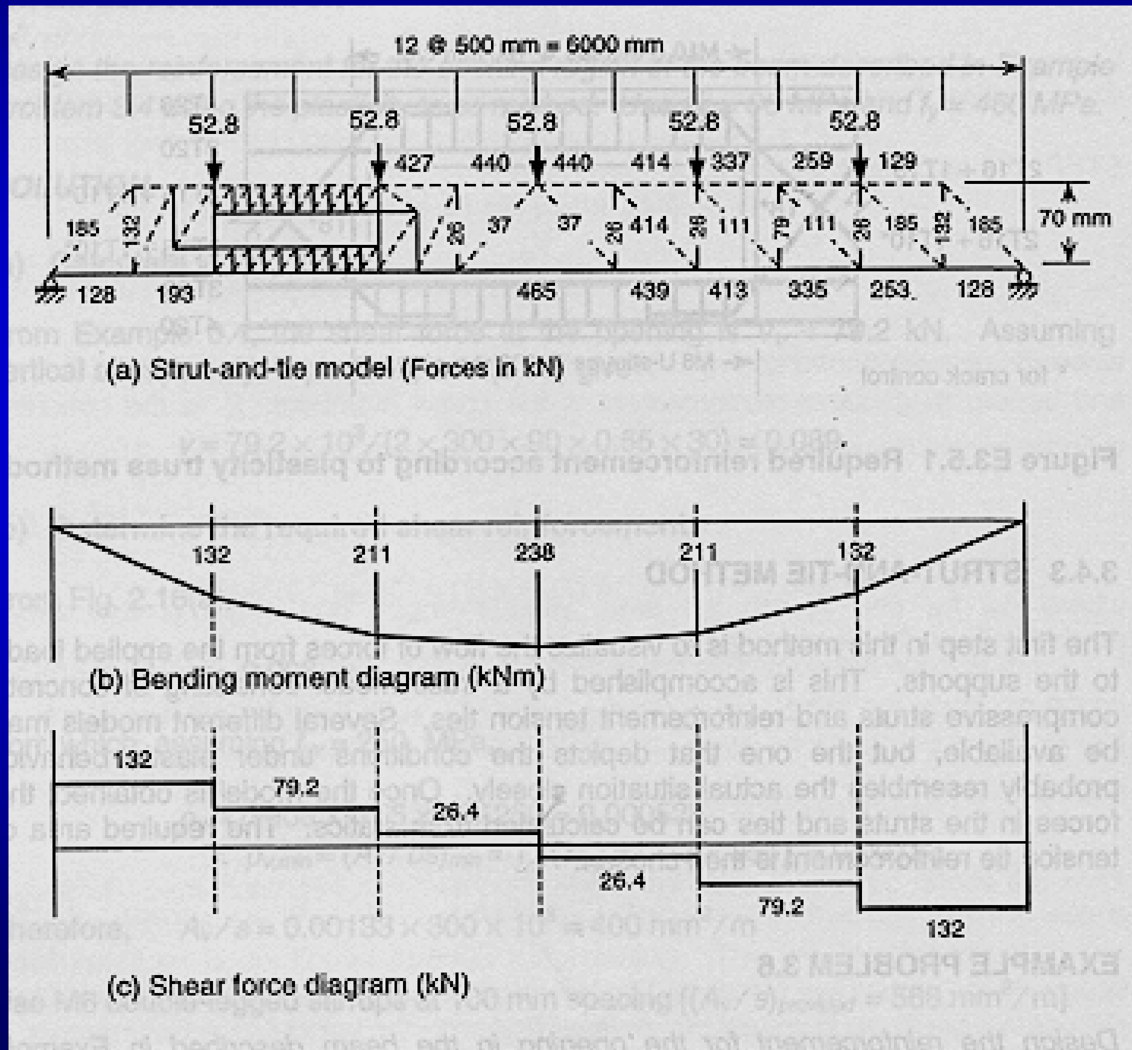
1. Idealize structure as comprising concrete struts and reinforcement ties, joined at nodes.
    - *follow stress path given by elastic theory; refine as necessary*
    - *equilibrium of strut and tie forces*
    - *deformation capacity of concrete*
    - *angle between struts and ties*

2. **Calculate forces in struts and ties from equilibrium.**

- *check stresses in struts (< $0.6f_{cd}$)*
- *detail reinforcement according to calculated tie forces*
- *check for anchorage of reinforcement*

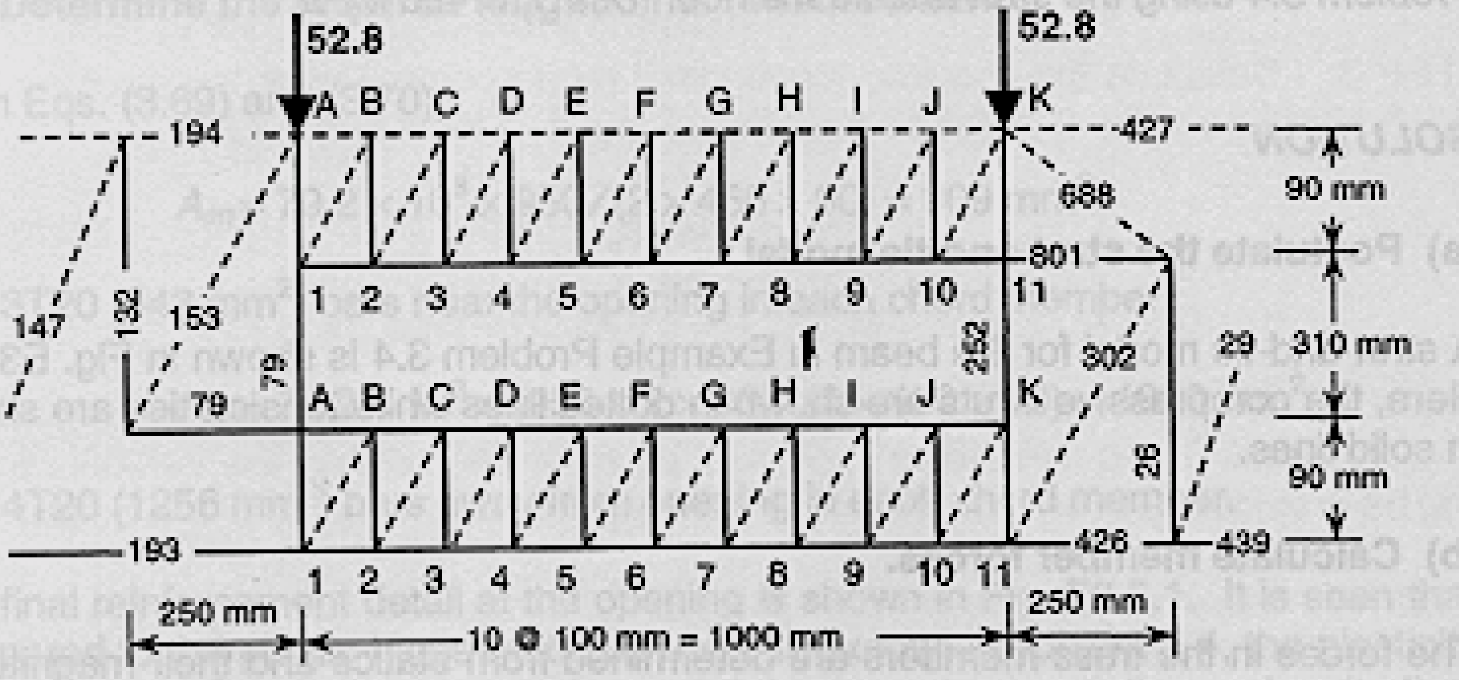
Solution by Strut-and-Tie Method

1. Postulate strut-and-tie model.



2. Calculate member forces.

Top chord takes 91% of total shear.
Stirrup force ≈ 3.3 times shear
(Model may be simplified.)



(d) Opening segment (Forces in kN)

3. Longitudinal reinforcement.

Top chord

Top steel: 1298 mm^2 (4T20)

Bot. steel: 1741 mm^2 (4T25)

Bottom chord

Top steel: 171 mm^2 (2T12)

Bot. steel: 590 mm^2 (2T20)

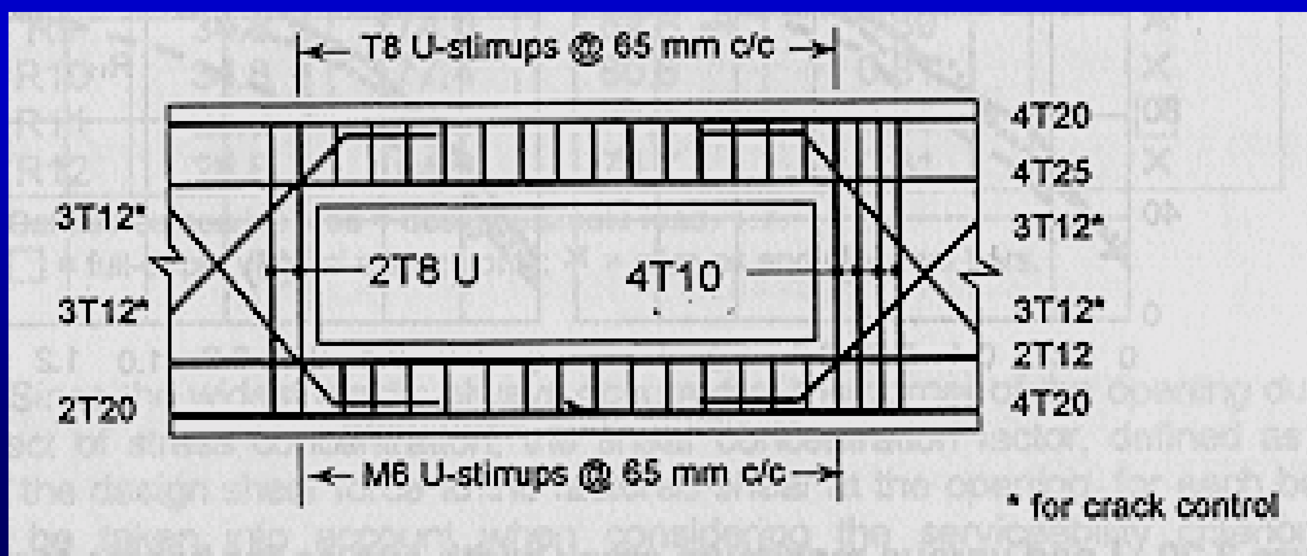
4. Transverse reinforcement.

Top chord

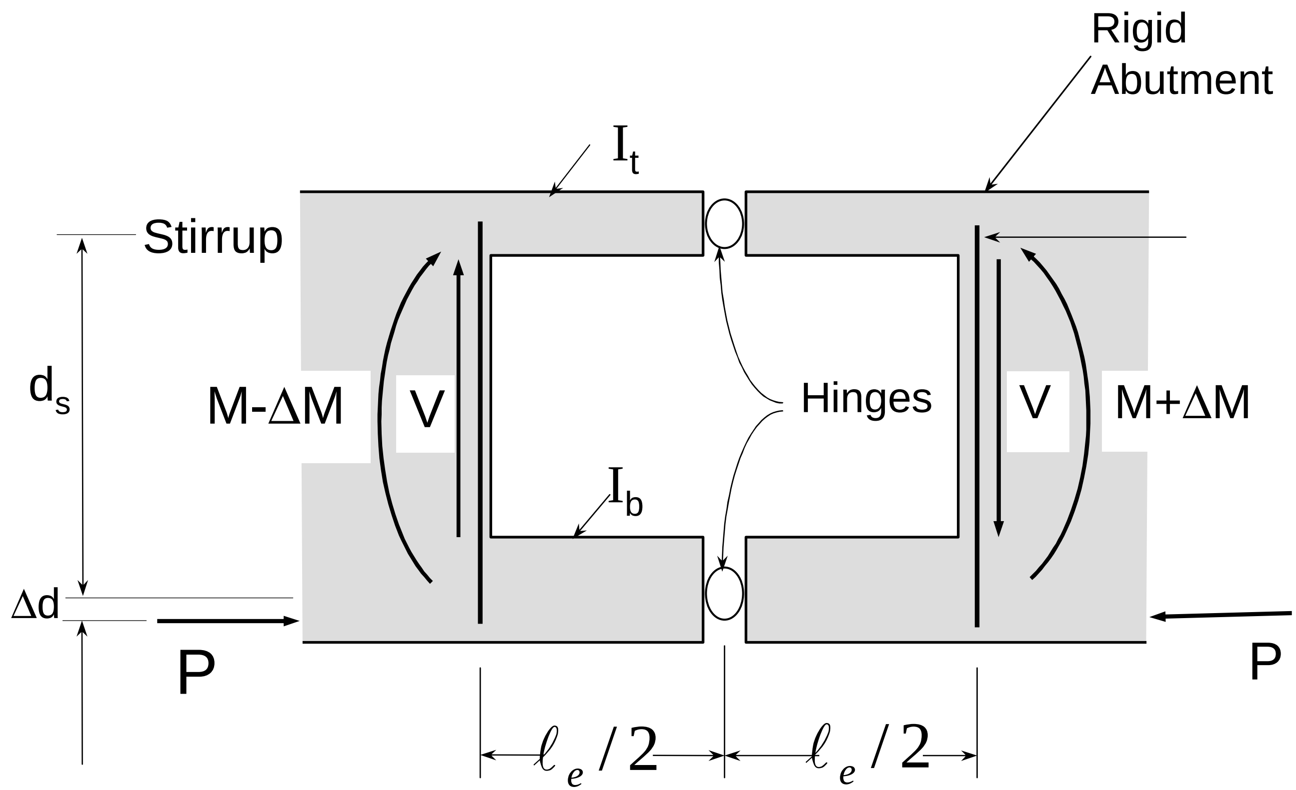
$A_{sv}/s = 1567\ mm^2/m$ (T8 @ 65 mm)

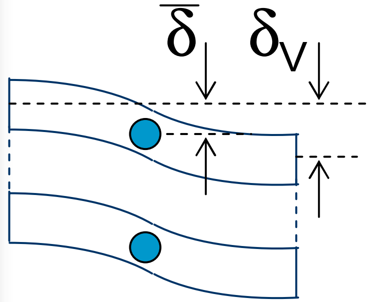
Bottom chord

$A_{sv}/s = 284\ mm^2/m$ (M6 @ 65 mm)



CALCULATION OF DEFLECTIONS





Relative displacement of hinge w.r.t. one end of opening due to V

(I_t based on gross concrete section; I_b based on a fully cracked section)

$$\bar{\delta} = \frac{V\left(\frac{\ell_e}{2}\right)^3}{3E_c\left(I_t + I_b\right)}$$

Relative displacement of one end of opening w.r.t. the other end

$$\delta_v = 2\,\bar{\delta} = \frac{V\,\ell_e^3}{12E_c\left(I_t + I_b\right)}$$

Midspan deflection of beam

$$\delta = \delta_w + \left(\delta_v\right)_{opening\,1} + \left(\delta_v\right)_{opening\,2} + \ldots\ldots$$

Calculate the midspan service load deflection of the beam described in the Example.

SOLUTION

Service load, $P_s = P_u / 1.7 = 52.8 / 1.7 = 31.1$ kN;

$V = 1.5\,P_s = 46.6$ kN.

Effective length of chord members, $\ell_e = (950 + 50)$

$= 1000$ mm.

Assume $I_t = I_g = 300 \times 180^3/12 = 146 \times 10^6\ \text{mm}^4$;

$I_b = 0.1 \times 146 \times 10^6\ \text{mm}^4 \approx 15 \times 10^6\ \text{mm}^4$

Also,

$E_c = 4730\sqrt{30} = 26 \times 10^3$ MPa

Hence,

$\delta_v = V\,\ell_e^3 / [12\,E_c(\,I_t + \,I_b)]$

$= 46.6 \times 10^{12} / [12 \times 26 \times (146 + 15) \times 10^9]$

$= 0.93$ mm

Midspan deflection, δ_w, of the beam is $11PL^3/144E^cI$, where I can be conservatively estimated as the moment of inertia of the beam at a section through the opening. That is,

$$I = 2 \times [300 \times 180^3 / 12 + 300 \times 180 \times 210^2]$$
$$= 5054 \times 10^6 \text{ mm}^4$$

As the span L is 6 m, therefore,

$$\delta_w = 11 \times 31.1 \times 10^3 \times (6000)^3 / [144 \times 26 \times 5054 \times 10^9]$$
$$= 3.91 \text{ mm}$$

Hence, the total midspan deflection is calculated as

$$\delta = \delta_v + \delta_w = 0.93 + 3.91 = 4.84 \text{ mm}$$
$$< L/360 = 16.7 \text{ mm}$$

Summary

Method	Crack control	Ultimate strength	Deflection
Mechanism approach	Corner reinft.: • factor of 2 • vert. stirrups ~25-50% • diagonal bars ~50-75%	*Rigorous:* • N, M from interaction diagram • V *Simplified:* • pt. of contra-flexure at midspan • $V_t/V_b = I_t/I_b$	Add deflection due to shear at each opening
Plasticity truss method (AIJ)	-	• check V_u against V_{truss} • A_{sn}, A_{sf} • shear reinft. over $d_{vs} \cot \phi_s$	
Strut-and-tie method	• check stress in tie against allowable value (eg. $f_y/2$)	• check stress in strut ($<0.6f_{cd}$) • reinforcement according to tie forces	